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MAGNETIC AMPLIFIERS

by

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MAGNETIC AMPLIFIERS

ABSTRACT

Direct current controlled transductors,now known as magnetic amplifiers,were used successfully by the Germans in the 1939-1945 war for the amplification and mixing of powers down to a microwatt.The desire to know more about their operations than has been given in various periodicals prompted this thesis.

An explanation of magnetic amplifier action is given and the waveforms of flux and output current predicted. The behaviour of a circuits containing a magnetic amplifier is then explained by the use of waveforms and a mathematical investigation is made.A use for a magnetic amplifier in an oscillator tank circuit is described.

Graphs of the characteristics of the magnetic amplifier and photographs of the waveforms are shown and the observed waveforms compared with the predicted ones. A sensitivity test on three circuits containing a magnetic amplifier is described and results given,showing that by placing a condenser across the supply windings increases the sensitivity considerably but decreases the power amplification.

The proper way to secure a thorough understanding of the properties of the magnetic amplifier by the use of an oscilloscope is discussed and the thesis concludes by

listing the advantages and disadvantages of a magnetic amplifier.

The mathematical approach to the solution of the magnetic amplifier problem using Frolich's equation for the magnetization curve is given in a special appendix.

John C.Tebby
U.B.C.
April,1951

ACKNOWLEDGEMENT

The author wishes to express his indebtedness to those who have assisted him towards the completion of this investigation; especially to Professor W.B. Coulthard for his expert guidance and encouragement.

Acknowledgement is also made to the British Columbia Telephone Company Limited whose scholarship made the year's work possible.

John C. Tebby

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MAGNETIC AMPLIFIERS

I INTRODUCTION

The purpose of this investigation is to develop a theory for a circuit containing RLC in series or in parallel with a nonlinear element such as a magnetic amplifier. Since little work has been done on the theory of the magnetic amplifier the initial investigation will concern the magnetic amplifier.

The terms magnetic amplifier, saturable reactor, inductor, transductor and amplistat have been used to denote iron-cored inductors in which the a.c. impedance is controlled by means of an auxiliary d.c. excitation. They have been known and applied for some fifty years being used mainly in the dimming of lights in theatres. They had also been used to a very limited extent in heavy current and light current power control. In the latter two cases since 1935 much progress has been made on their design particularly in England.

During the 1939-1945 war considerable progress was made in Germany on the development of single phase amplifiers for servo applications, while in Sweden the main work seems to have been on three phase inductors suitable for large power outputs. In the German long range rocket an inductor weighing three pounds with a core of Mumetal maintained the alternator frequency to within ± 1%. Applications were also found in auto-pilots, servo mechanisms and blind approach systems. From various accounts it would appear that although German technicians obtained excellent results they gained most of their knowledge through trial and error not by the use of theoretical design principles.

Many articles dealing with the magnetic amplifier have been published however the majority of them are non-mathematical.None,this author included,has been able to devise a non-linear method for the solution of the magnetic amplifier. A general solution by non-linear methods does not exist,however step-by-step integration yields a particular solution applying only to a particular curve.For this reason all of the authors have eliminated the solution by non-linear methods in favor of an approximate solution by linear methods.In order to obtain the solutions by linear methods assumptions must be made.The published articles have for the most part made the following approximations to convert the non-linear equations to linear ones:

1. The BH curve can be approximated to be made up of three straight lines,a region of constant permeability folloowed by complete saturation.
2. The reactance of the transductor with zero control is so large compared with the resistance of the load that the difference in phase between the applied voltage and that across the transductor is negligible.
3. The rectifiers are perfect and the back leakage is zero.

With the preceding approximations it has been possible to predict in advance the characteristics of a particular magnetic amplifier.This has contributed greatly to the progress that has been made in the past few years.

II THEORY OF THE MAGNETIC AMPLIFIER

The most important component of a magnetic amplifier is the inductor which consists of two a.c.chokes with a superimposed d.c.winding as shown in Fig.1.In Fig.1 an inductor is shown where the d.c.windings are connected in series opposition.

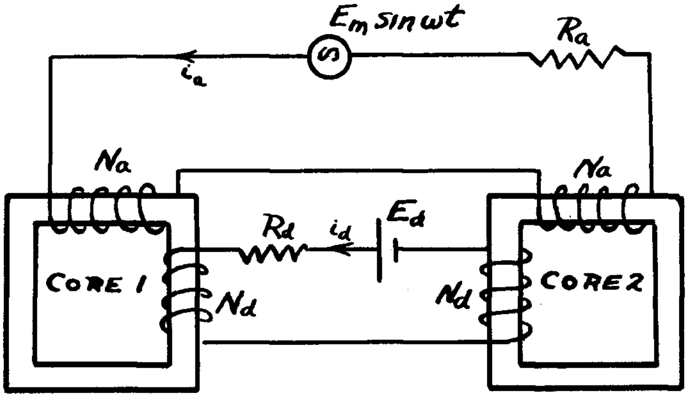


Fig.1

As the inductor d.c.windings are connected in series opposition the superimposed d.c.excitation will cause one core to saturate in the negative flux direction and later in the cycle the other core will saturate in the positive flux direction as shown in Fig.2.

The voltage applied to the a.c.windings during the period 0 to α is positive and the flux in both cores increases until eventually core 1 becomes saturated positively and remains so until the voltage enters the next half cycle.

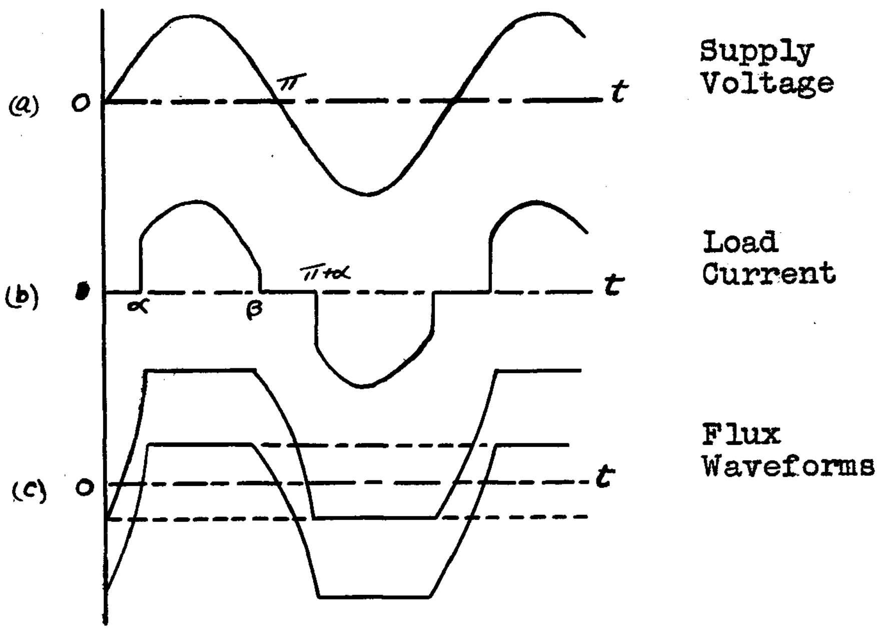


FIG.2

Core 1 has no impedance when saturated and increases the current through core 2 which also becomes saturated and therefore the whole of the supply voltage between α and β is transferred to the load resistance.Between 0 and α the output voltage across the load resistance is small because being unsaturated the transductor reactance is large compared to the load resistance.During the next half cycle the action is repeated with the difference that it is core 2 that saturates in the negative flux direction.

Because the applied voltage is sinusoidal the flux waveforms must be sinusoidal and each core must contribute half the flux i.e.the supply voltage will divide evenly between each core.When we add d.c.excitation one flux will be increased and the other will be decreased because of the reversal of the windings.The flux densities may be represented by the following two equations:

$$B_0 \sin\omega t + B_m$$
$$B_0 \sin\omega t - B_m$$

Neglecting hysteresis effects, the magnetization curve for Stalloy is represented by the following equation;

$$H = aB + bB^3 + cB^5 + dB^7 \text{ ampere turns per cm.}$$

When we have a material requiring a low number of ampere-turns to saturate it we have to add 9th. or 11th. powers and the equation is altered. By putting equation 1 into equation 3 we get the excitation for core 1 as obtained by Hauffe[1]L

$$\begin{aligned} h_1 = {} & aB_m + bB_m^3 + cB_m^5 + \tfrac{3}{2}bB_0^2B_m + \tfrac{15}{8}cB_0^4B_m + 5cB_0^2B_m^3 \\ & + (aB_0 + \tfrac{3}{4}bB_0^3 + \tfrac{5}{8}cB_0^6 + 3bB_0B_m^2 + \tfrac{15}{2}cB_0^3B_m^2 + 5cB_0B_m^4)\sin\omega t \\ & - (\tfrac{3}{2}bB_0^2B_m + \tfrac{5}{2}cB_0^4B_m + 5cB_0^2B_m^3)\cos 2\omega t \\ & - (\tfrac{1}{4}bB_0^3 + \tfrac{5}{16}cB_0^5 + \tfrac{5}{2}cB_0^3B_m^2)\sin 3\omega t \\ & + (\tfrac{5}{8}cB_0^4B_m)\cos 4\omega t + (\tfrac{1}{16}cB_0^5)\sin 5\omega t \end{aligned}$$

The a.c. excitation is represented by the equation

$$\begin{aligned} i_a N_a = {} & (aB_0 + \tfrac{3}{4}bB_0^3 + \tfrac{5}{8}cB_0^5 + 3bB_0B_m^2 + \tfrac{15}{2}cB_0^3B_m^2 \\ & + 5cB_0B_m^4)\sin\omega t - (\tfrac{1}{4}bB_0^3 + \tfrac{5}{16}cB_0^5 + \tfrac{5}{2}cB_0^3B_m^2)\sin 3\omega t \\ & + \tfrac{1}{16}cB_0^5 \sin 5\omega t \end{aligned}$$

and thus the output current will contain a third harmonic which accounts for the peaky shape of the waveform.

The control circuit excitation consists of a steady state and a second, fourth etc. harmonic with the harmonics being contributed by the supply voltage. We have therefore:

$$I_{av} = \frac{2}{\pi}\frac{1}{N_a}[B_0(a + 3bB_m^2 + 5cB_m^4) + B_0^3(\tfrac{3}{2}b + \tfrac{20}{3}cB_m^2) + \tfrac{8}{15}cB_0^5$$

The curve of inductor output against control current shows that we have a symmetrical curve about zero input current however there is a small magnetizing current which varies with supply voltage.With two transductors it is desirable to have identical characteristics and this is accomplished in two ways,by self-excitation or by bias excitation.

The mathematical theory of the magnetic amplifier is the next step and this will be worked out for an amplifier with a resistance load,with a condenser in series,finally for a condenser in parallel with the supply windings of the amplifier.

We will not work out a mathematical theory for the oscillator illustrated in the last part of the next section since it is a standard oscillator circuit.The purpose of mentioning the oscillator will appear in the text of that section.

III MATHEMATICAL INVESTIGATION

One of the approximations to convert the non-linear equations to linear ones was that the BH curve could be approximated by three straight lines-a region of constant permeability followed by complete saturation. Thus we will assume a characteristic of the shape shown in Fig.3.

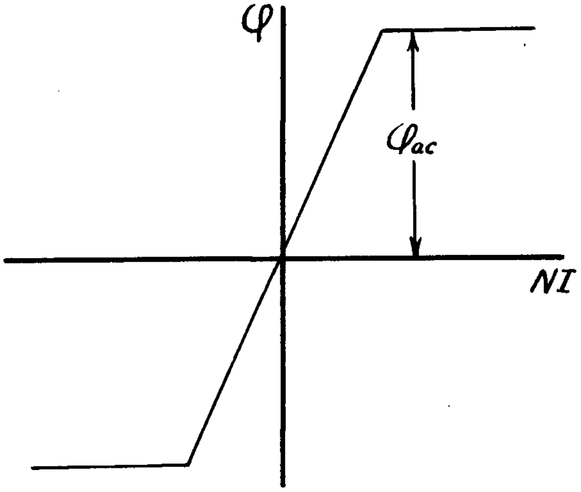


Fig.3

The first circuit to be analyzed will be the one of Fig.4 where we have two cores with no additional windings as in the conventional amplifier.

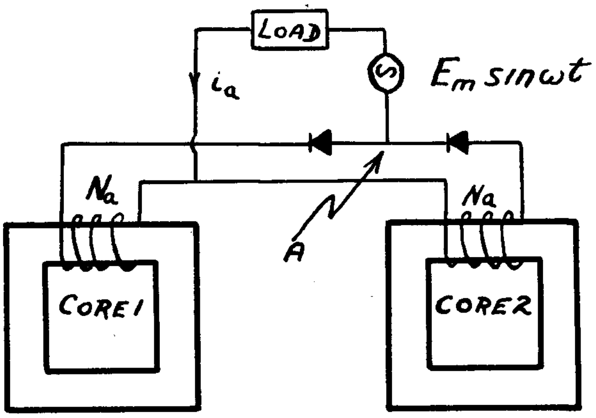


Fig.4

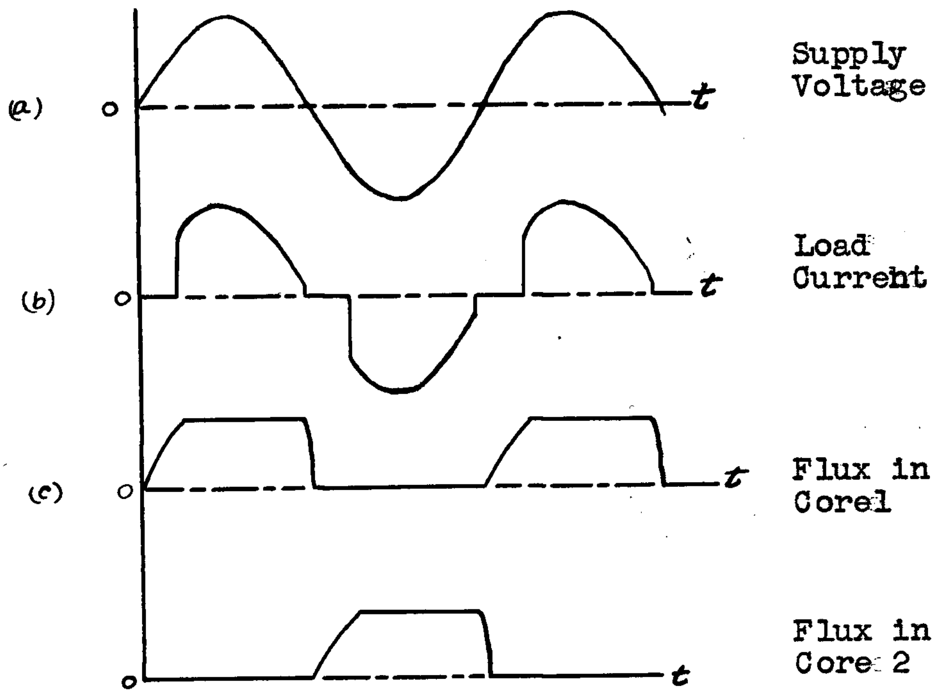


Fig.5

Considering Fig.5a,as the voltage at point A of Fig. 4 increases sinusoidally from 0 to point a,the value of i_a is very low because of the high reactance of core 1,therefore the voltage drop-E_{load}-across the resostance is low and

$$\frac{N_a}{10^8}\frac{d\varphi_1}{dt} = E_m \sin\omega t$$

is shown in Fig.5c.Between 0 and a, the flux waveform will be sinusoidal as $\frac{d\varphi_1}{dt} = \frac{N_a}{10^8} E_m \sin\omega t$ .At a, core 1 saturates

therefore $\frac{d\varphi_1}{dt} = 0$ and $E_{LOAD} = E_m \sin\omega t$

This saturated condition exists until the voltage drop across the supply winding is no longer high enough to maintain the saturation current and therefore $\frac{d\varphi_1}{dt} \neq 0$ .When this occurs the reactance of core 1 becomes very high and again approximately the whole of the supply voltage is across core 1 from b, to π .During the next half cycle core 2 follows the conditions previously outlined for core 1 and core 1 is inert.

Consider next the circuit of Fig.6 which is the same as that of Fig.4 with a d.c.control winding. This d.c. flux alters the BH curve as shown in Fig.7

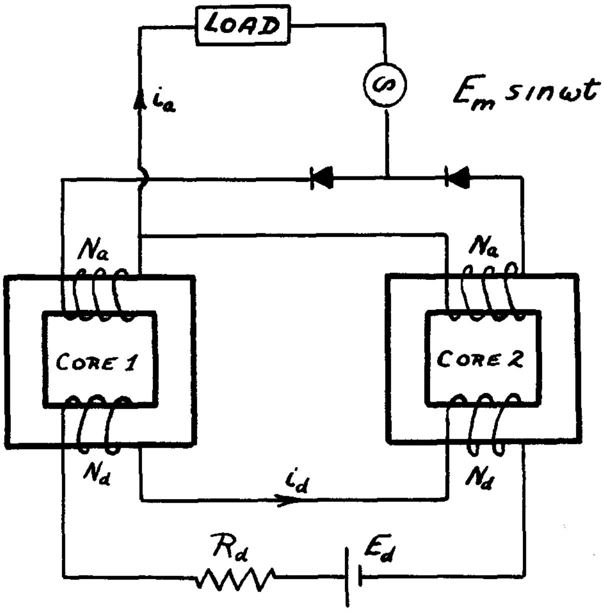


Fig.6

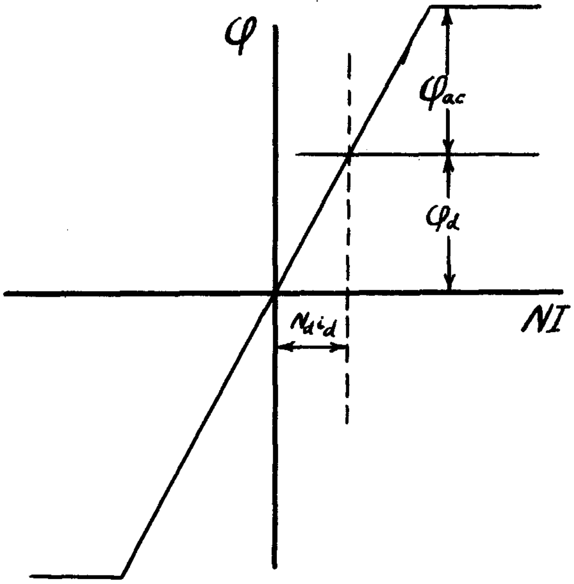


Fig.7

As shown in Fig.7 the required a.c.magnetizing current for saturation will be less than for the previous

The circuit of Fig.6 will have the equation as follows:

$$E_{LOAD} + \frac{N_a}{10^8}\frac{d}{dt}[\varphi_1] = E_m \sin\omega t$$

$$\varphi_1 = \varphi_{ac} + \varphi_d$$

As the voltage at point A of Fig.6 increases sinusoidally from 0 to point a_1 the value of i_a is very low and thus E_{load} will be small and therefore

$$\frac{N_a}{10^8}\frac{d}{dt}[\varphi_1] = E_m \sin\omega t$$

Since φ_d is a constant, φ_1 will consist of a sinusoidal flux and a steady value flux as shown in Fig.8c.

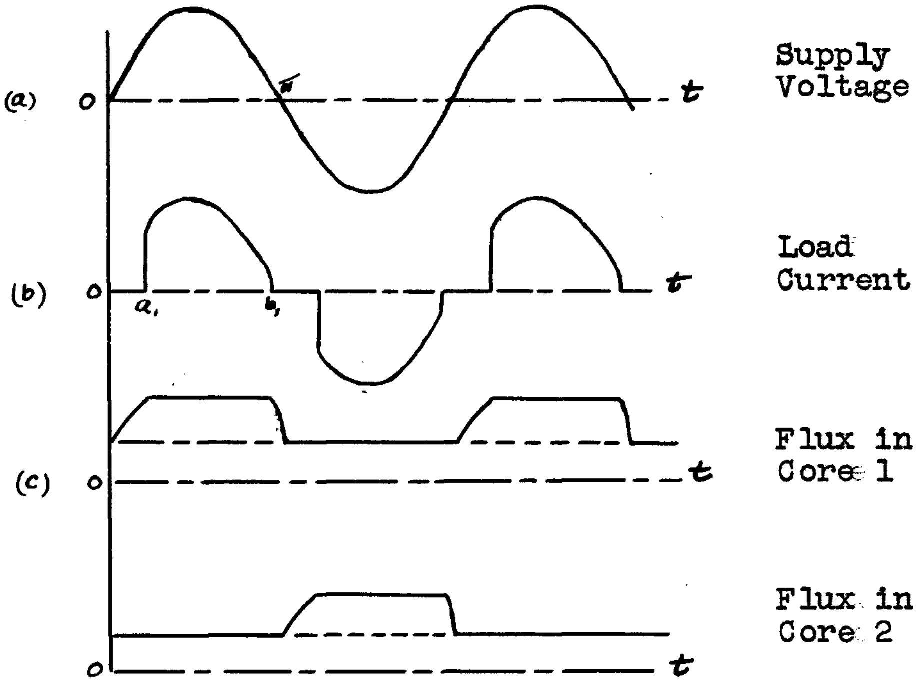


FIG 8

At point a_1, the core saturates and thus φ_1 is a constant. Since $\frac{d\varphi_1}{dt} = 0$, $E_{LOAD} = E_m \sin\omega t$ which is shown in Fig.8b. As the supply voltage passes from a_1 to b_1 the core will remain saturated, however at b_1, the supply voltage is not high enough to maintain saturation conditions i.e. $\frac{d\varphi_1}{dt} \neq 0$ and therefore i_a is very low and the flux decreases to the steady d.c. value.

For the next half cycle the process is repeated on core 2.

When i_d is negative the BH characteristic will be altered as in Fig.9 and the a.c.magnetizing current required for saturation will be larger than previously. The waveforms will be shown in Fig.10.

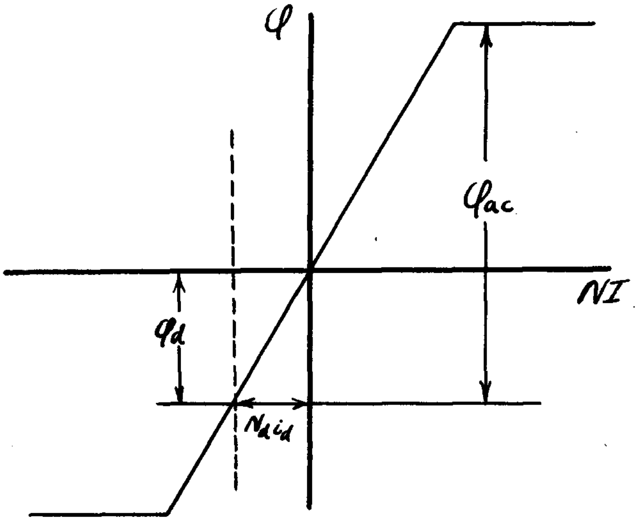


Fig.9

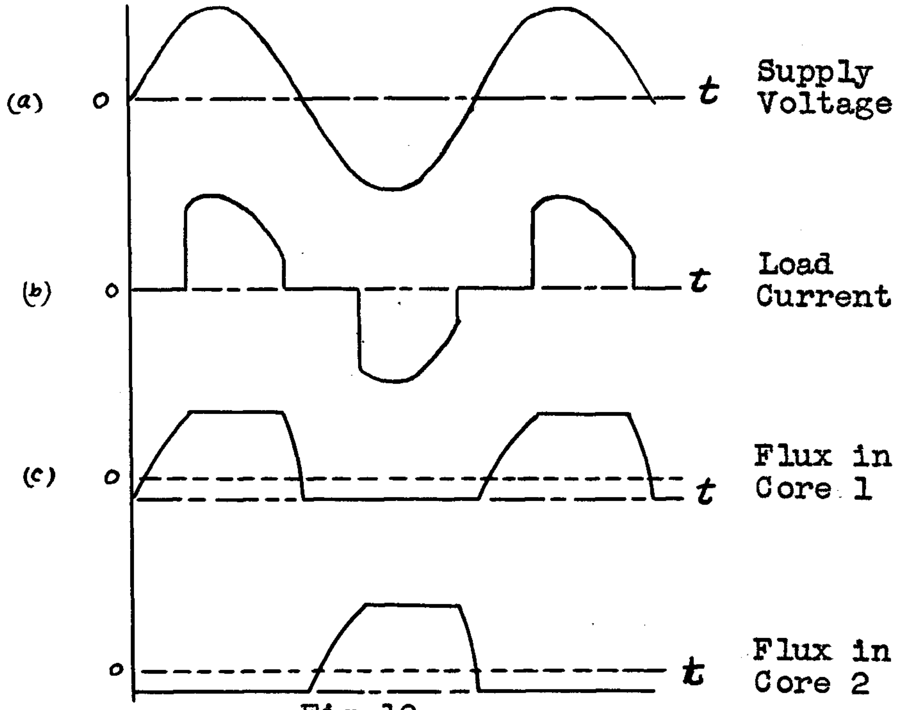


Fig.10

Note the length of the saturated portion of the flux wave and how it has decreased from the length when i_d was positive. The cores will saturate later in the cycle and unsaturate earlier in the cycle.For this condition the equations are:

$$E_{LOAD} + \frac{N_a}{10^8}\frac{d\varphi_1}{dt} = E_m \sin\omega t$$

$$\varphi_1 = \varphi_{ac} - \varphi_d$$

IV CIRCUIT THEORY

Consider a circuit as Fig.11 where we have a condenser and a resistance inserted for the load.

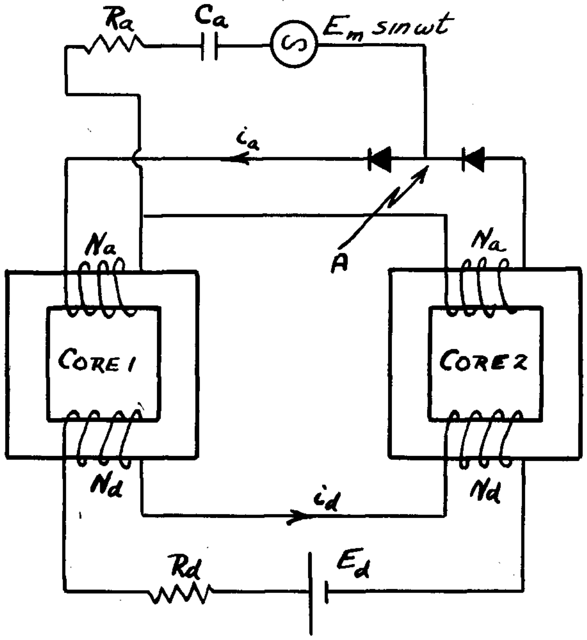


Fig.11

The general equations are:

$$R_a i_a + \frac{1}{C_a}\int i_a\,dt + \frac{N_a}{10^8}\frac{d}{dt}(\varphi_1) = E_m \sin \omega t$$

The load voltage will be approximately zero for the period 0 to a_1 of Fig.8a and for that period

$$\frac{N_a}{10^8}\frac{d}{dt}(\varphi_1) = E_m \sin \omega t$$

and the flux will have the waveform previously shown in Fig. 8c. From a_1 to b_1 the core is saturated and therefore $\frac{d\varphi_1}{dt} = 0$ which gives us $R_a i_a + \frac{1}{C_a}\int i_a\,dt = E_m \sin \omega t$

the steady state solution for which is given in Appendix A and is

$$i_a = \frac{E_m \omega}{R_a}\left[\omega \sin \omega t - \frac{1}{C_a R_a}\sin(90^\circ + \omega t)\right]\left[\frac{1}{\frac{1}{C_a^2 R_a^2} + \omega^2}\right]$$

From b_1 to π the flux waveform will be sinusoidal as in Fig.8c.

Consider next the magnetic amplifier with a resistance load. A condenser is connected across the a.c. windings. For the circuit see Fig.12.

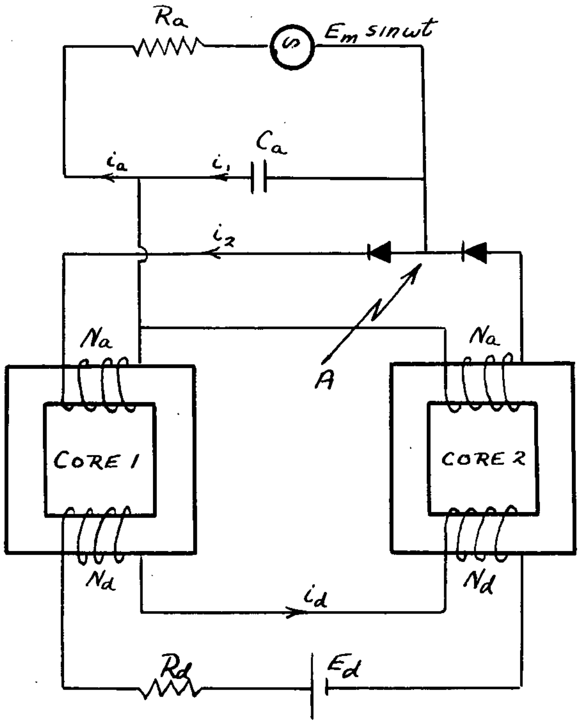


Fig.12

The general equations are: $i_a = i_1 + i_2$

$$R_a i_a + \frac{N_a}{10^8}\frac{d}{dt}(\varphi_1) = E_m \sin\omega t$$

$$\frac{1}{C_a}\int i_1\,dt = \frac{N_a}{10^8}\frac{d}{dt}(\varphi_1)$$

As the voltage at point A (Fig.12) increases from 0 to a_1 (Fig.13a) the reactance of core 1 is very high and therefore i_2 small, however a current will flow through C_a and thus a voltage drop will occur across the load. For this condition the circuit reduces to a series RC circuit and the equations are: $i_a = i_1$

$$R_a i_1 + \frac{1}{C_a}\int i_1\,dt = E_m \sin\omega t$$

the steady state solution for which is (Appendix A)

$$i_1 = \frac{E_m\omega}{R_a}\left[\frac{\omega\sin\omega t - \frac{1}{C_aR_a}\sin(90^\circ+\omega t)}{\frac{1}{C_a^2R_a^2}+\omega^2}\right]$$

Let the voltage across the core be E if the voltage across the core required for the saturation current to be developed is E_s. From 0 to a_1 $E_m \sin\omega t = R_a i_a + E$. At a_1 the supply voltage is such that E is equal to E_s, therefore the core saturates. Under this condition the negligible impedance of the core being in parallel with the capacitance reduces the reactance of the parallel circuit to a low value and therefore the value of E is low and so

$$\frac{N_a}{10^8}\frac{d\varphi_1}{dt} \to 0$$

$$\frac{N_a}{10^8}\frac{d\varphi_1}{dt} + R_a i_a \to R_a i_a = E_m \sin\omega t$$

From a_1 to b_1 the core remains saturated until the supply voltage reaches such a value that the voltage across the core is no longer able to maintain the core in saturation. From b_1 to π the supply voltage is divided between R_a and C_a. For the next half cycle the waveforms are repeated except that core 2 will be involved instead of core 1. It will be noted that as i_1 and i_2 are added the resultant waveform of Fig.13f is almost a sine wave. Since this is also a representation of the load voltage it will approximate a sine wave of the same form as the current wave.

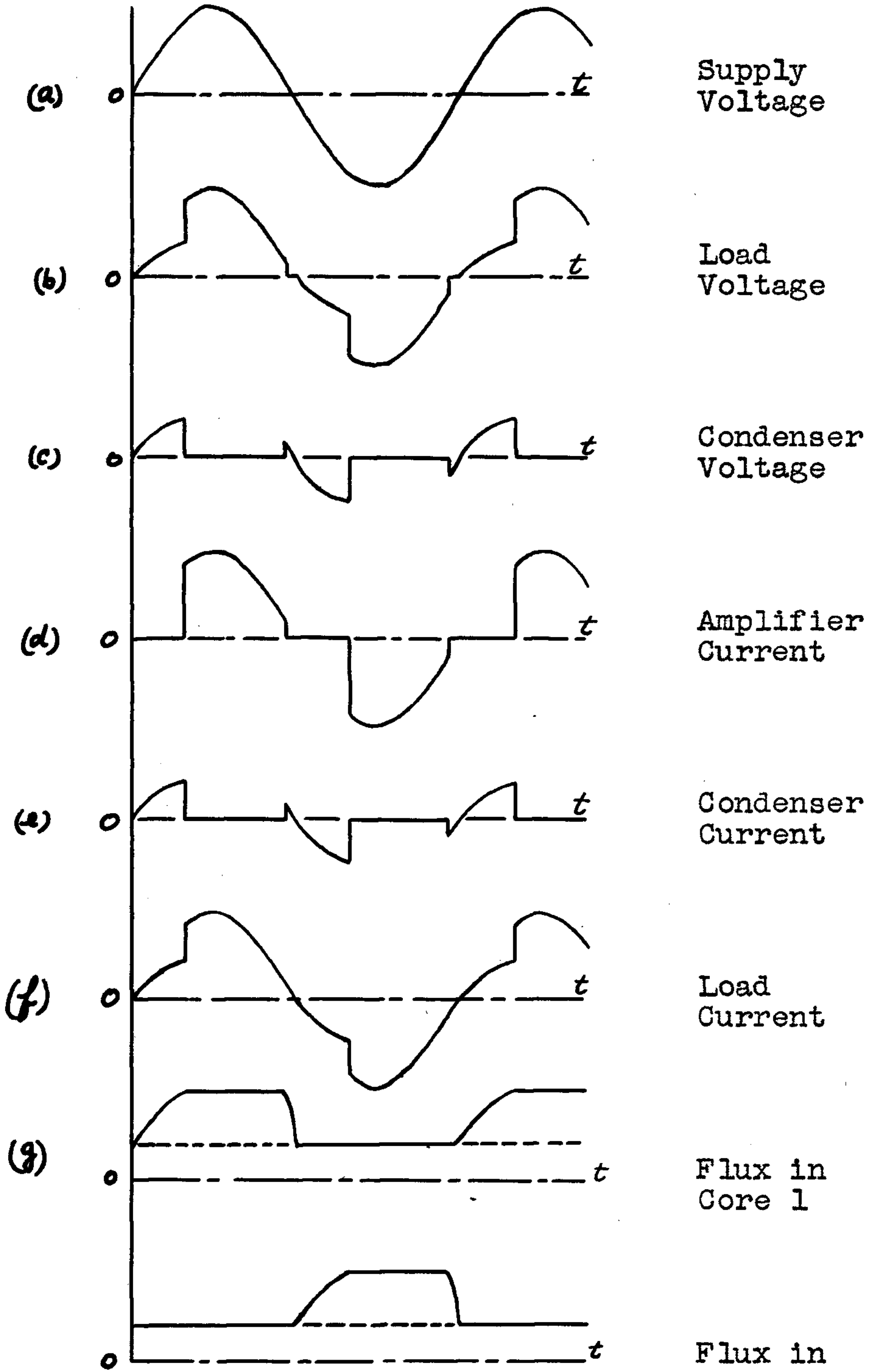

(a)
o
t
Supply Voltage
(b)
o
t
Load Voltage
(c)
o
t
Condenser Voltage
(d)
o
t
Amplifier Current
(e)
o
t
Condenser Current
(f)
o
t
Load Current
(g)
o
t
Flux in Core 1
o
t
Flux in Core 2

Fig.13

With the circuit of Fig.11 it is possible to have a condition where there is no load voltage nor will the core be saturated, for this to occur the reactance of the condenser will have to be high. By carefully choosing C_a we can have a condition where the core will be saturated for only a fraction of a cycle thus giving a sharp current pulse. This results when the voltage across the amplifier is high enough to cause saturation and desaturation only when the supply voltage is near its peak.

Consider next the circuit of Fig.12. With this circuit it is possible to have a condition where there is zero output yet the core is saturated. The current through the capacity will be between 90° and 180° out of phase with the current flowing through the saturated core winding, thus a condition can exist where these two currents cancel, hence the load current is zero. The reactance of the capacity in this circuit will have to be high otherwise the current flowing through it will be very much larger than the current through the amplifier. With that condition the amplifier will be unable to exert any control over the output.

IVC OSCILLATOR USING AN IRON CORED INDUCTANCE

Consider next the circuit illustrated in Fig.14. This is a tuned grid oscillator circuit with a magnetic amplifier as the inductance in the circuit. The purpose of using the magnetic amplifier in this circuit is to find out how the inductance varies as the d.c. control winding is excited up to saturation. We will also have from the results an indication of the shape of the BH curve.

Let $f=\frac{1}{2\pi\sqrt{LC}}$ for this oscillator, then $L=\frac{1}{\omega^2 C}$. While this is not the correct formula the approximation will suffice for our purpose.

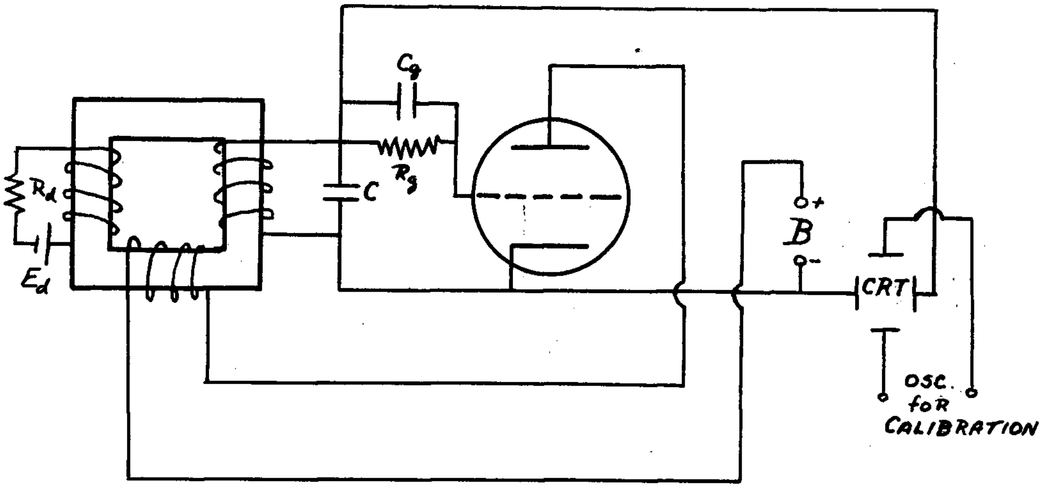


Fig.14

V APPARATUS

The magnetic amplifier used in the investigation was a Vickers Electric Educational Magnetic Amplifier(see footnote).The control current was measured with a Weston d.c.milliameter,the load current with a Weston a.c.ammeter and the load voltage with a General Radio vacuum tube voltmeter.

For the oscillator circuit of Fig.14 a 6J5 tube was used.The oscillator was calibrated on a Cossor Oscilloscope by comparing its frequency with that from a Hewlett-Packard oscillator.

The photographs of the waveforms were taken on a Westinghouse six element oscillograph using Verichrome # 122 film.

Various values of resistances and condensers were required,both fixed and variable,as well as sources of direct current and a 110 volt,60 cycle supply.

For additional information on this amplifier see the "Laboratory Manual of Specifications,Instruction Notes and Experiments for the Educational Magnetic Amplifier" published by the Vickers Electric Division,1815 Locust St.,St.Louis 3,Missouri,U.S.A.

VI PROCEDURE

Tests were made on the circuits of Figs.6,11 and 12.The graphs and photographs following are the results of these tests.The data on the graphs and photographs states what circuit was used and what was varied.

For Fig.16 the circuit of Fig.14 was connected up and various values of control current were used to vary the frequency.For each value of control current the frequency was measured.This was accomplished by placing the oscillator output on one set of the Cossor oscilloscope deflection plates and the output of the Hewlett-Packard oscillator on the other set of deflection plates of the same beam.The Hewlett-Packard oscillator frequency was then altered until both frequencies were the same as indicated by the circle,oval or straight line waveform being motionless,the shape of the waveform depending on the phase displacement and voltage difference between the two outputs.

A test comparing the sensitivities of the circuits of Figs.6,11 and 12 was made.R_a and C_a were 150 ohms and 10 microfarads respectively.By using the graphs of Figs.15 and 23 the mid point of the steepest portion of each graph was found.The control current was then varied 2 ma.around this value,i.e. ± 1 ma.,and the change in output current measured.

A test to find the time lag of the amplifier in responding to a change in control current was made.The control current was abruptly changed from 0 ma.to 13 ma. and a photograph made while this change was taking place.

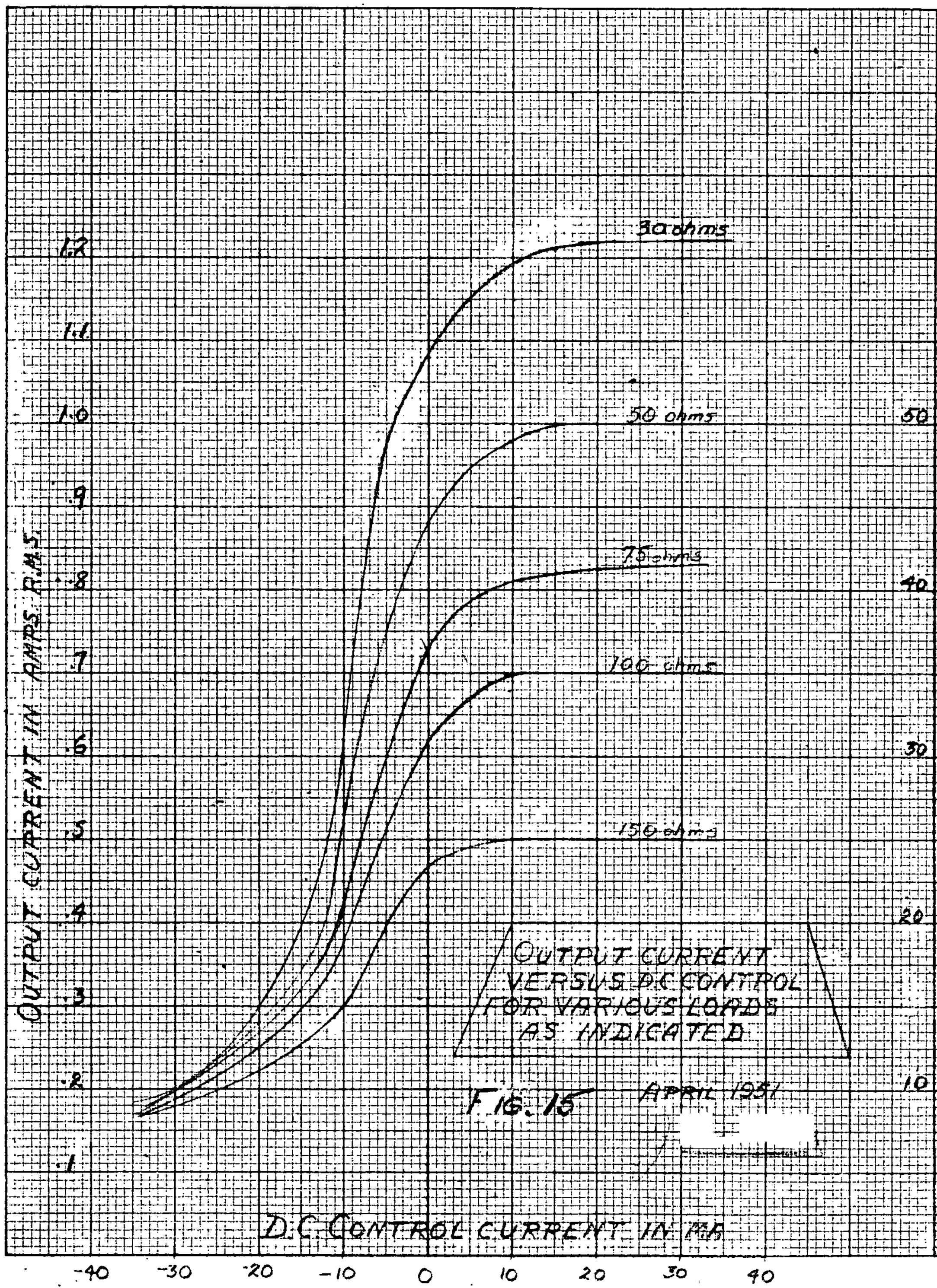
VII EXPERIMENTAL RESULTS

The graphs and photographs following this page show the results of the various tests made on the magnetic amplifier.Other tests were carried out with intermediate values and also values outside the ranges indicated however the results obtained were very similar to those for the ones shown and thus they were not included.

The sensitivity tests on the circuits of Figs.11, 6 and 12 gave the following results:

1. For the circuit of Fig.6,a change in control current from -5ma.to-7ma.gave a change in load current from .40 amps.to .36 amps.
2. For the circuit of Fig.11,a change in control current from -14 ma.to - 16 ma.gave a change in load current from .22 amps.to .14 amps.
3. For the circuit of Fig.12,a change in control current from -11 ma.to -13 ma.gave a change in load current from .18 to .32 amps.

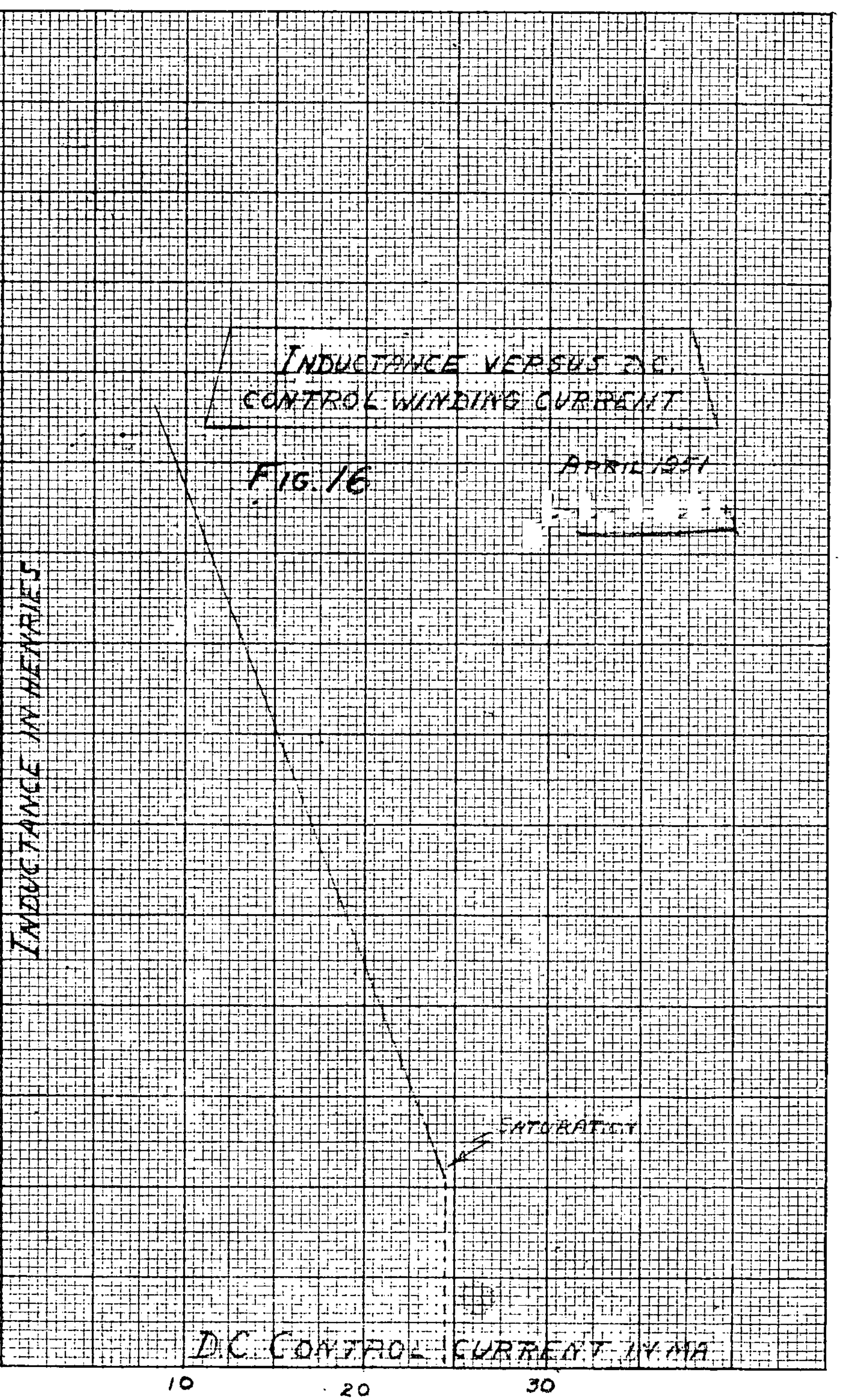
The photograph,Fig.31,made to find the time lag of the amplifier in responding to a change in control current shows that the amplifier required one cycle of the 60 cycle timing wave,i.e.1/60,second,to respond fully to the change.



OUTPUT CURRENT VERSUS D.C. CONTROL FOR VARIOUS LOADS AS INDICATED

FIG. 15

APRIL 1951



INDUCTANCE VERSUS D.C. CONTROL WINDING CURRENT

FIG. 16

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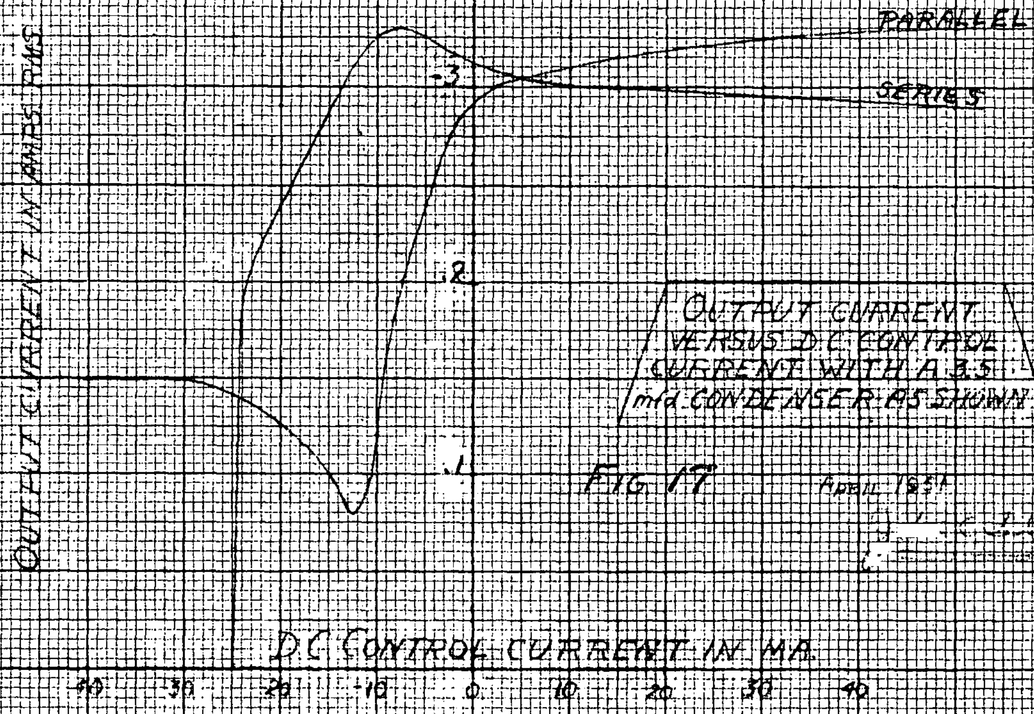
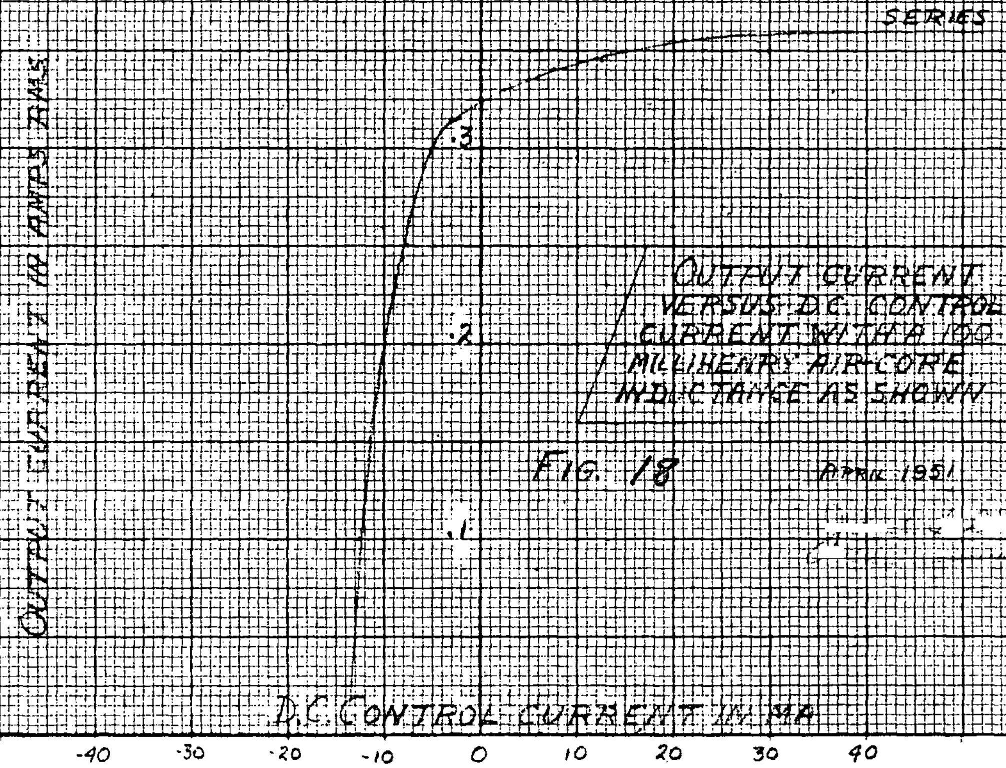


OUTPUT CURRENT VERSUS D.C. CONTROL CURRENT WITH A 3.5 mfd CONDENSER AS SHOWN

FIG. 17

APRIL 1951



OUTPUT CURRENT VERSUS D.C. CONTROL CURRENT WITH A 100 MILLIHENRY AIR-CORE INDUCTANCE AS SHOWN

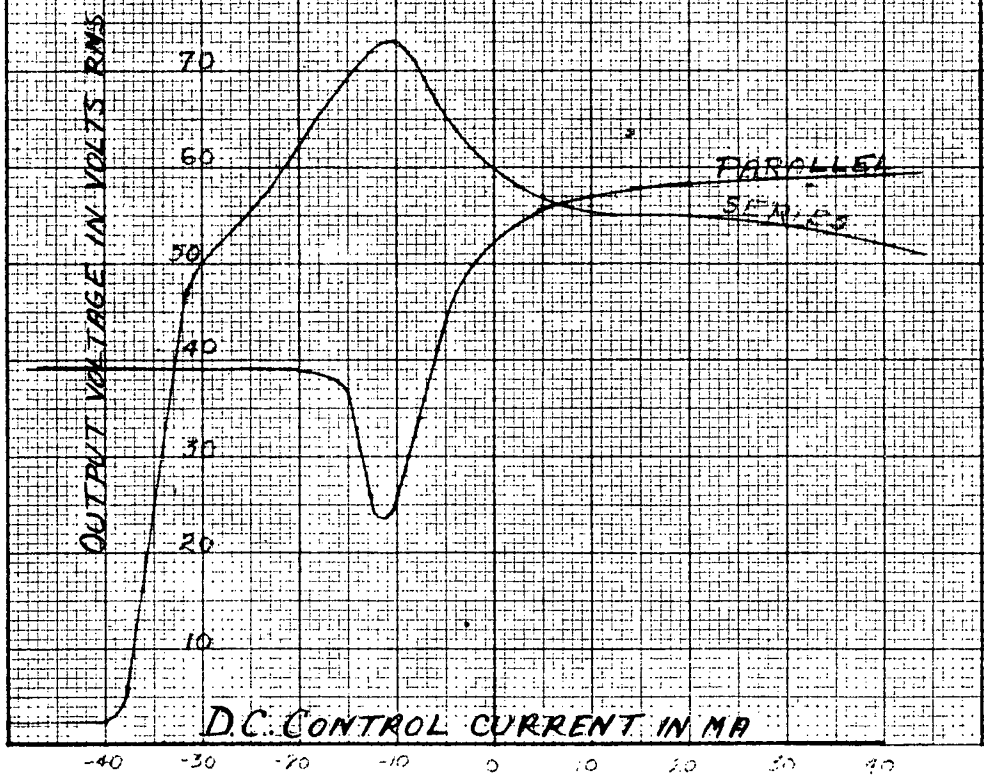
FIG. 18

APRIL 1951

OUTPUT VOLTAGE
VERSUS D.C. CONTROL
CURRENT WITH A 6.5 mfd
CONDENSER AS INDICATED

FIG. 19

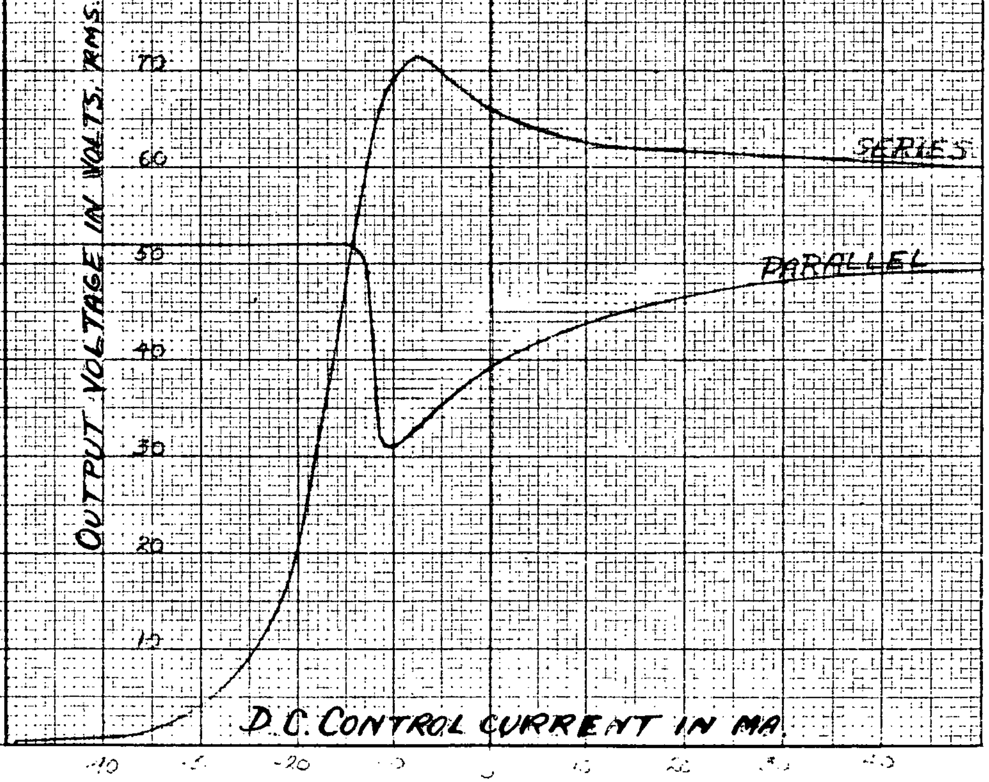
APRIL 1957



OUTPUT VOLTAGE
VERSUS D.C. CONTROL
CURRENT WITH A 10 mfd
CONDENSER AS INDICATED

FIG. 20

APRIL 1957

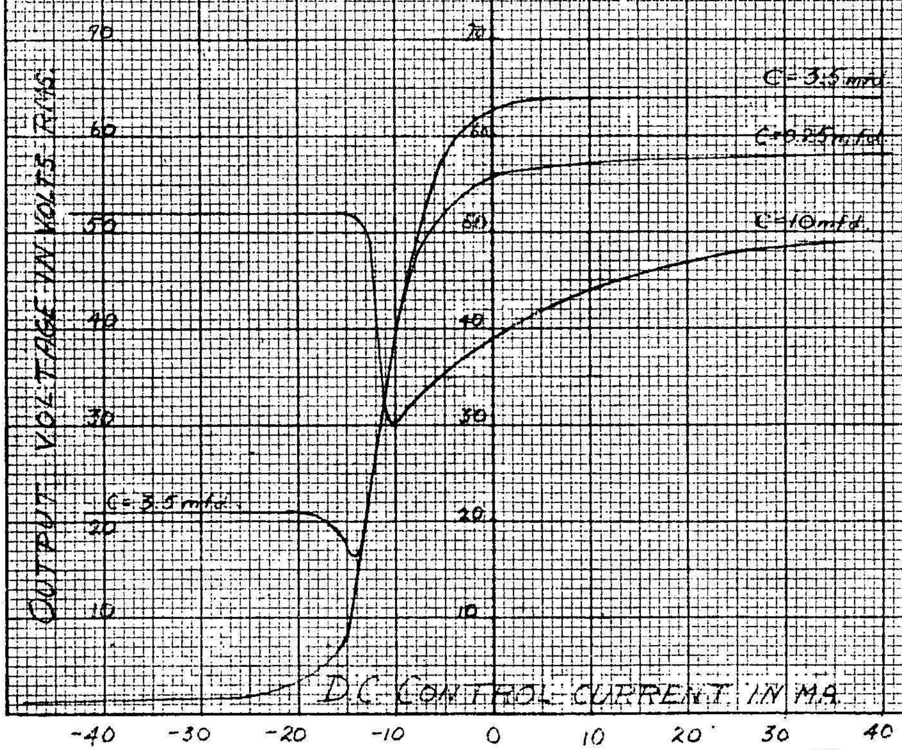


OUTPUT VOLTAGE
VERSUS D.C. CONTROL
CURRENT WITH VARIOUS
VALUES OF CAPACITY IN
PARALLEL

FIG. 21

APRIL 1951

[redacted]

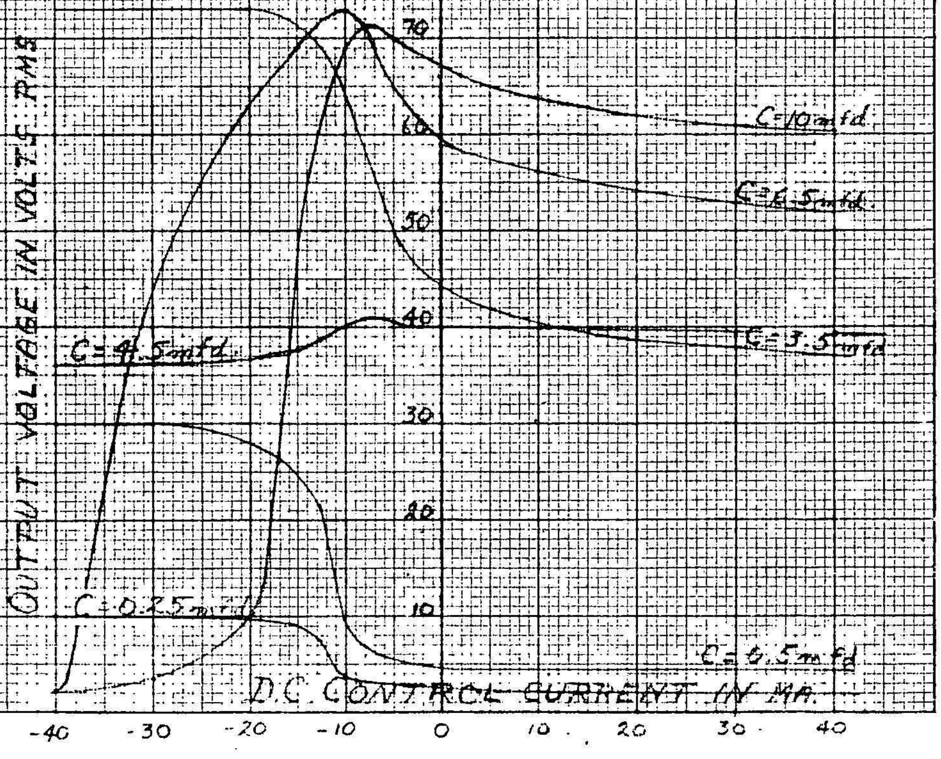


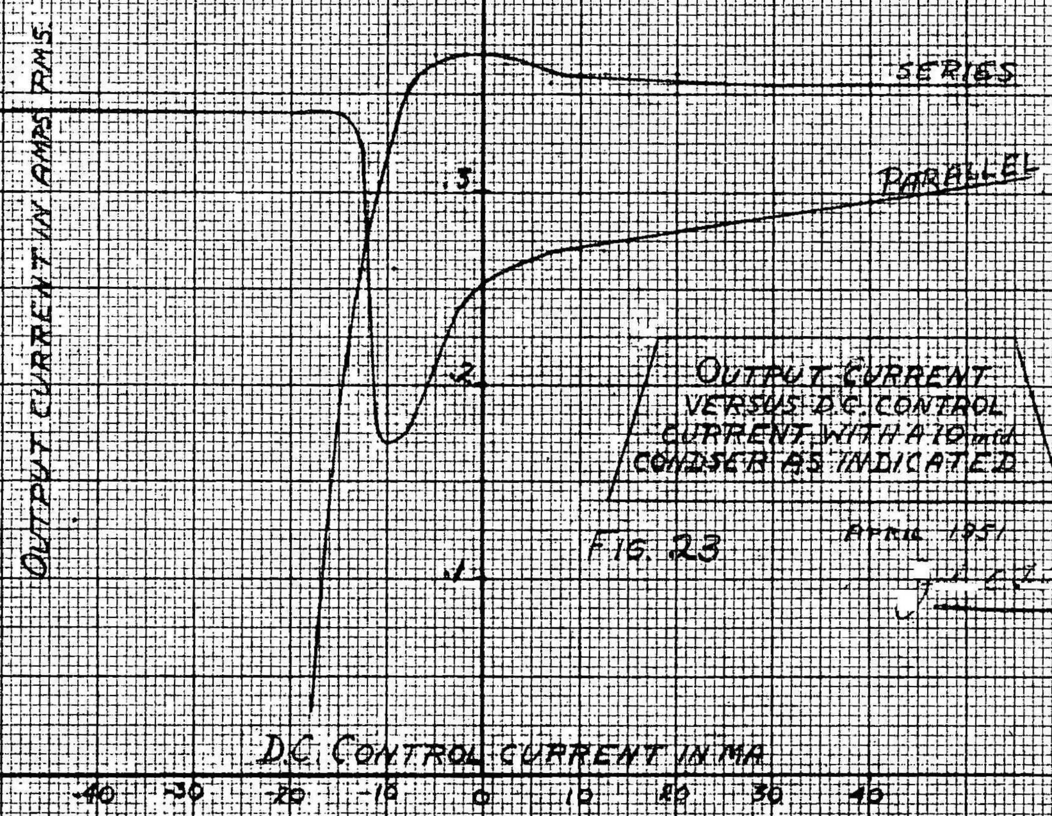
OUTPUT VOLTAGE
VERSUS D.C. CONTROL
CURRENT WITH VARIOUS
VALUES OF CAPACITY IN
SERIES

FIG. 22

APRIL 1951

[redacted]

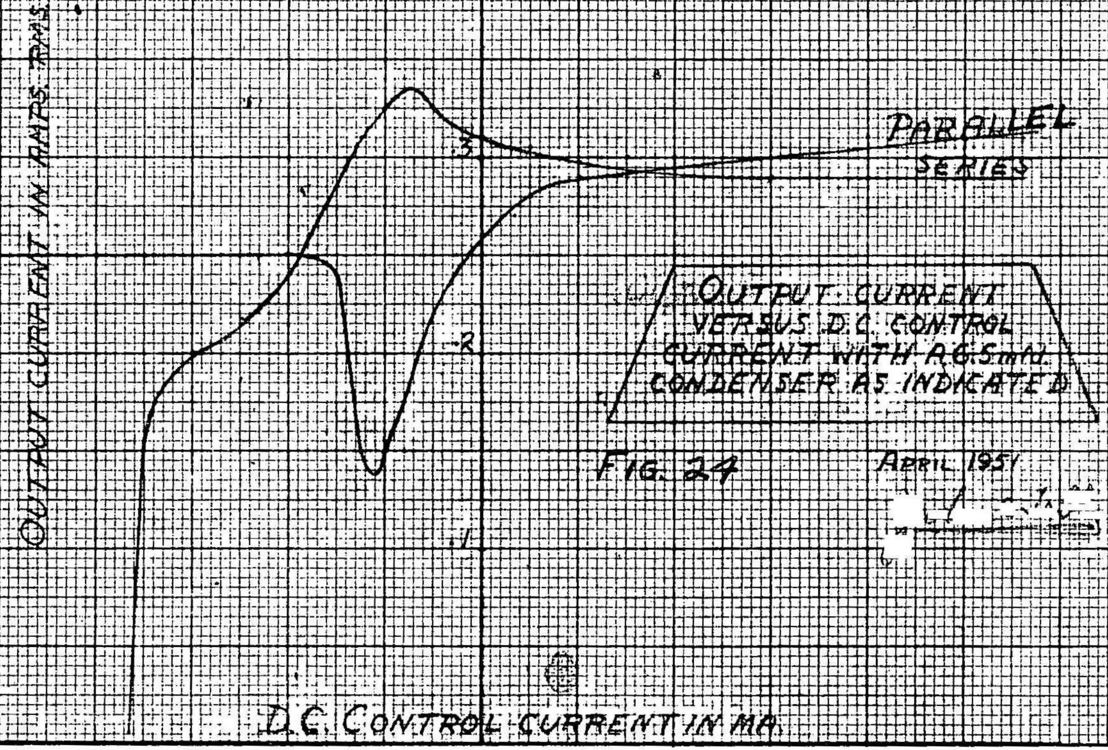




OUTPUT CURRENT VERSUS D.C. CONTROL CURRENT WITH A 10 mfd CONDSER AS INDICATED

FIG. 23

APRIL 1951



OUTPUT CURRENT VERSUS D.C. CONTROL CURRENT WITH A 6.5 mfd CONDENSER AS INDICATED

FIG. 24

APRIL 1951

Waveform showing load current with a resistance load

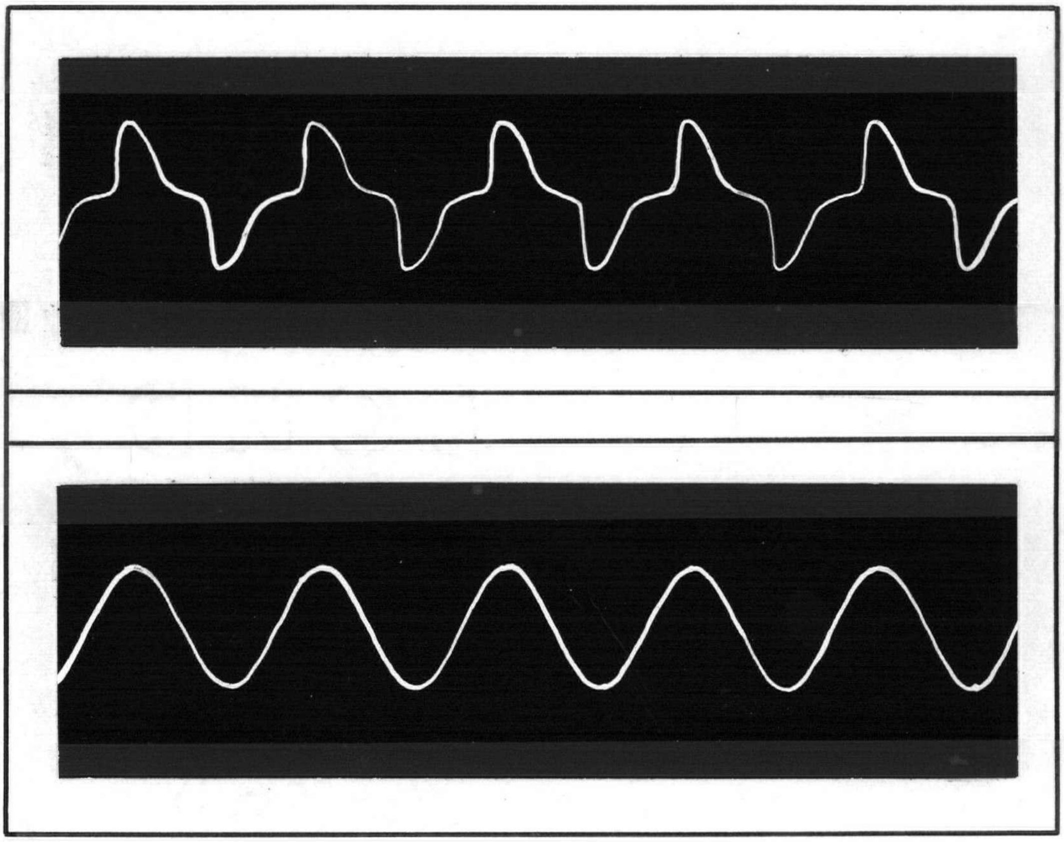


Fig. 25

<u>Top Waveform</u> Control current - -5 ma.
Load current - .415 amps.rms.
Load voltage - 78 volts rms.
Load resistance - 150 ohms.

<u>Bottom Waveform</u> Sine wave of 60 cycle supply frequency

Waveform of load current with a 100 ohm resistance as load

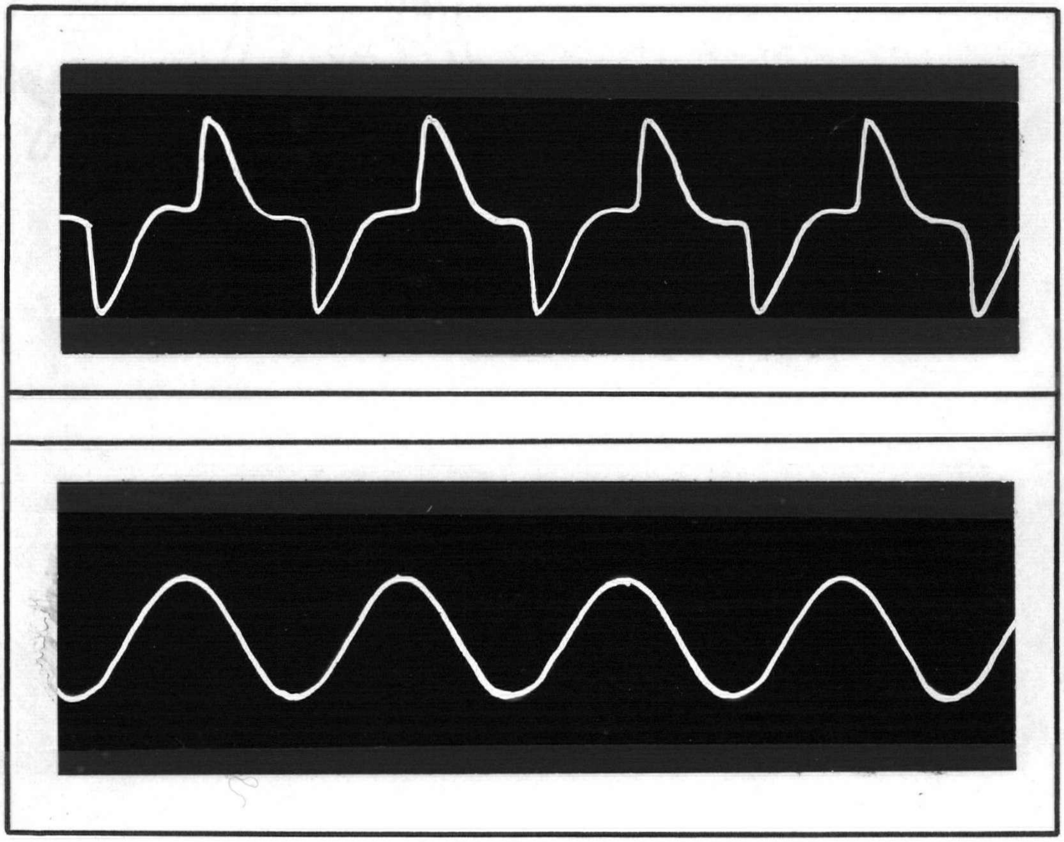


Fig. 26

<u>Top Waveform</u>
Control current - 0 ma.
Load current - .58 amps.rms.
Load voltage - 71 volts rms.
Load resistance - 100 ohms

<u>Bottom Waveform</u> 60 cycle sine wave of supply frequency

Waveform showing load current with a condenser in parallel

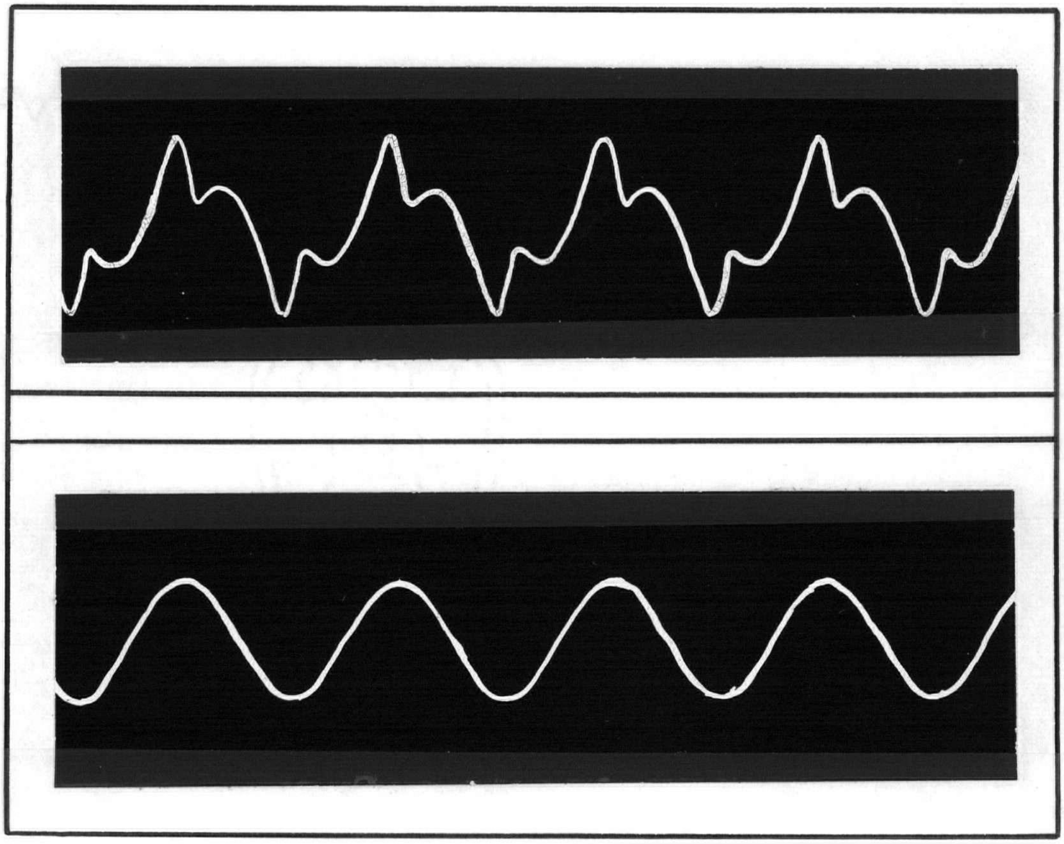


Fig. 27

Top Waveform Control current - 0 ma.
Load current - .37 amps.rms.
Load voltage - 73 volts rms.
Load resistance - 150 ohms
Condenser of 7.5 mfd.in parallel
Current in condenser branch - .31 amps.rms.
Current in amplifier branch - .54 amps.rms.

Bottom Waveform Sine wave of 60 cycle supply frequency

Waveform of load current with a condenser in parallel

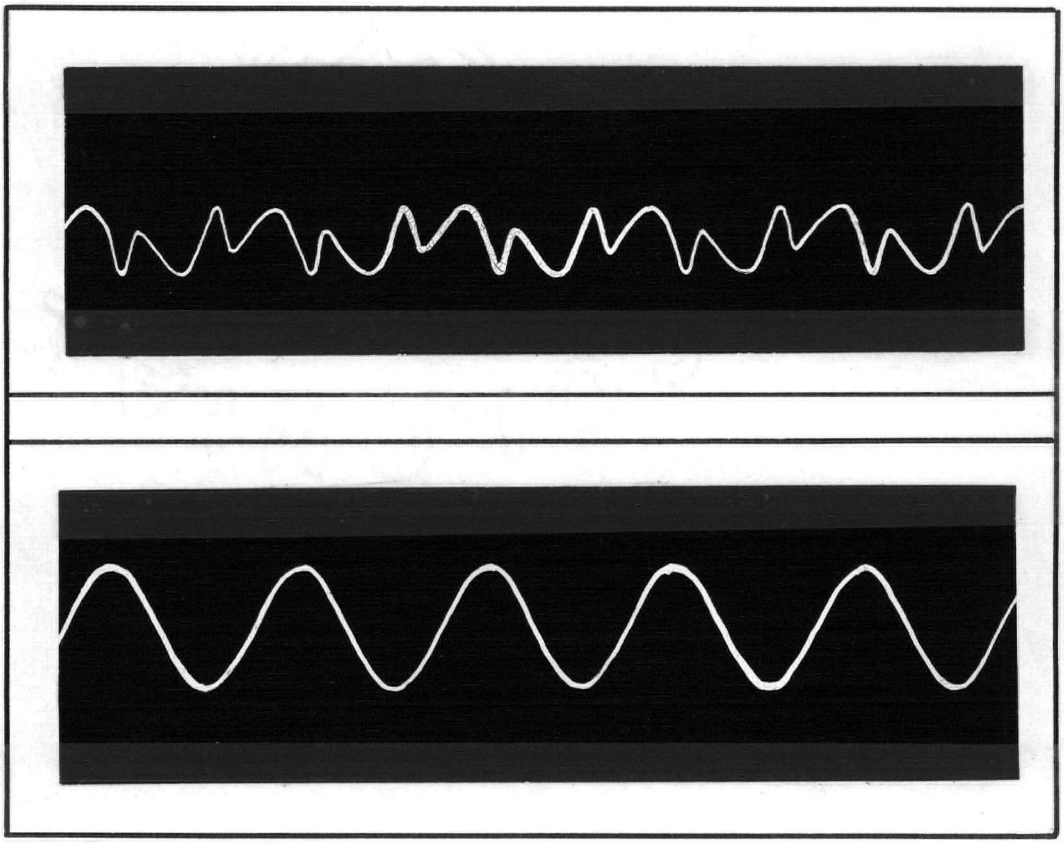


Fig. 28

Top Waveform Control current - -10 ma.
Load current - .19 amps.rms.
Load voltage - 31 volts rms.
Load resistance - 150 ohms.
Condenser of 7.5 mfd.in parallel
Current in condenser branch - .36 amps.rms.
Current in amplifier branch - .36 amps.rms.

Bottom Waveform Sine wave of 60 cycle supply frequency

Waveform of load current with a condenser in parallel

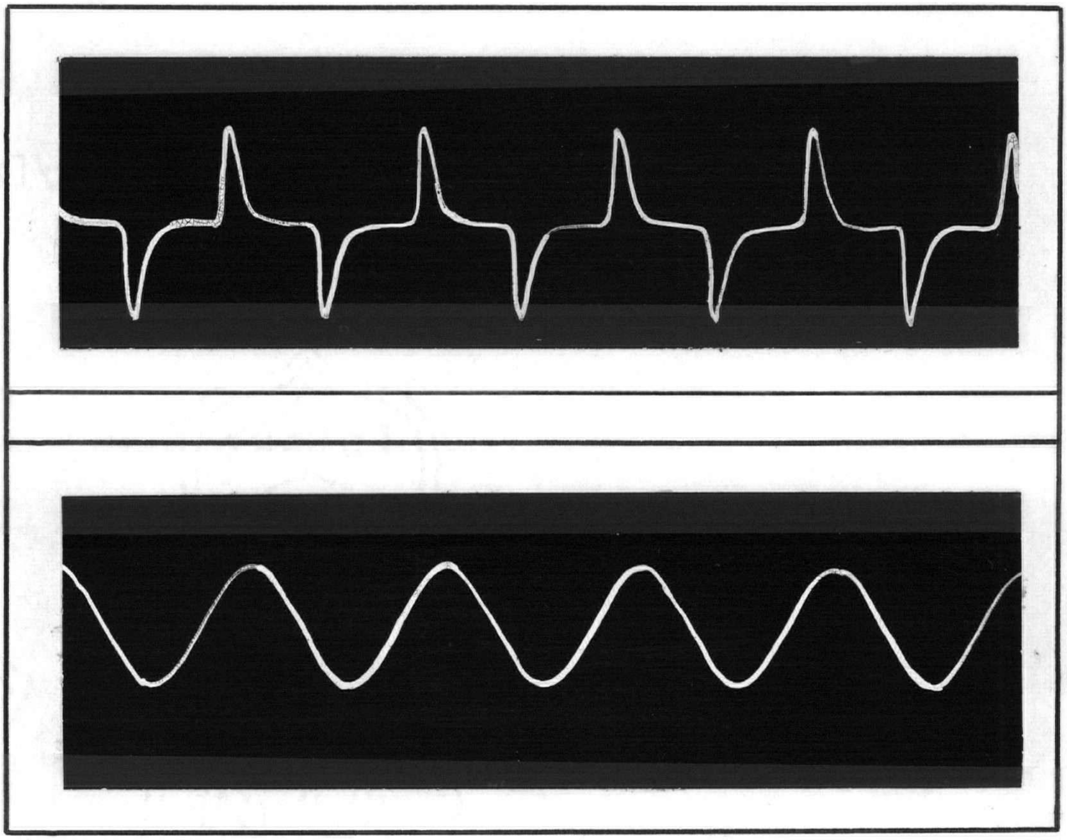


Fig. 29

Top Waveform Control current - -25 ma.
Load current - .34 amps.rms.
Load voltage - 94 volts rms.
Load resistance - 150 ohms
Condenser of 6.5 mfd.in parallel

Bottom Waveform Sine wave of 60 cycle supply frequency

Waveform of load current with a condenser in series

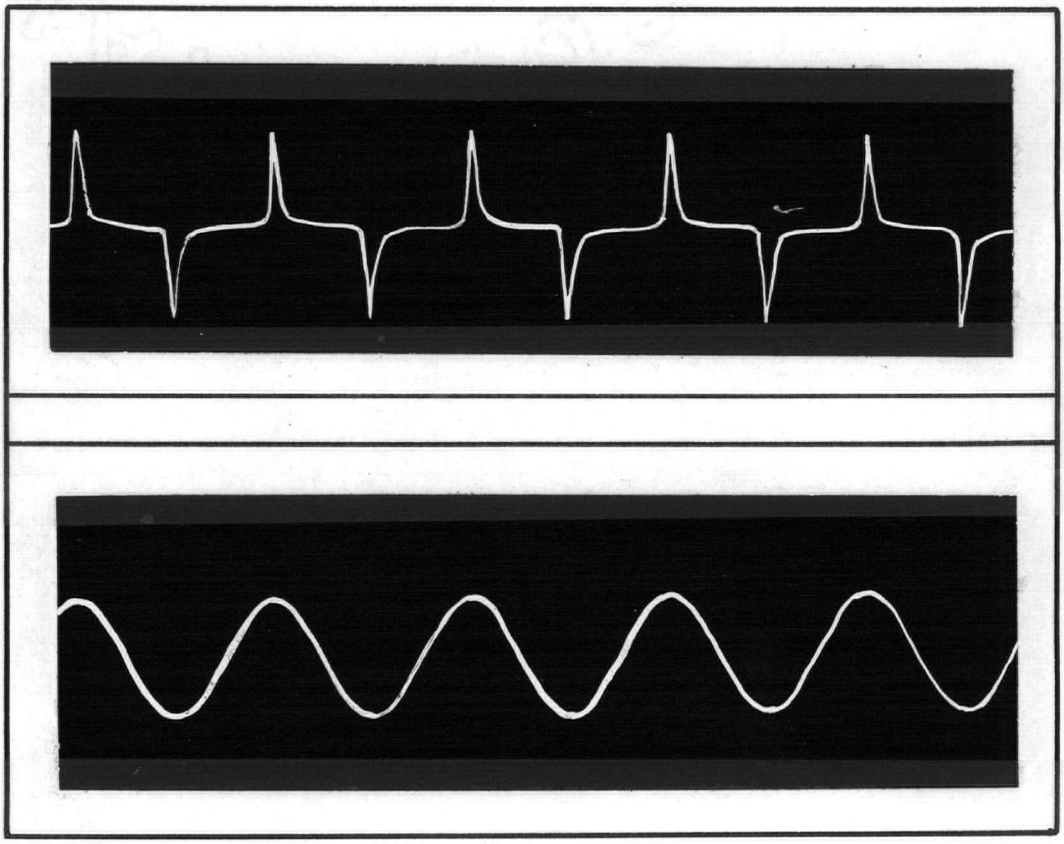


Fig. 30

<u>Top Waveform</u>
Control current - -25 ma.
Load current - .27 amps.rms.
Load voltage - 91 volts rms.
Load resistance - 150 ohms
Condenser of 2.5 mfd.in series

<u>Bottom Wave form</u> 60 cycle sine wave of supply frequency

Wavefroms showing the time lag in the amplifier response to a change in control current

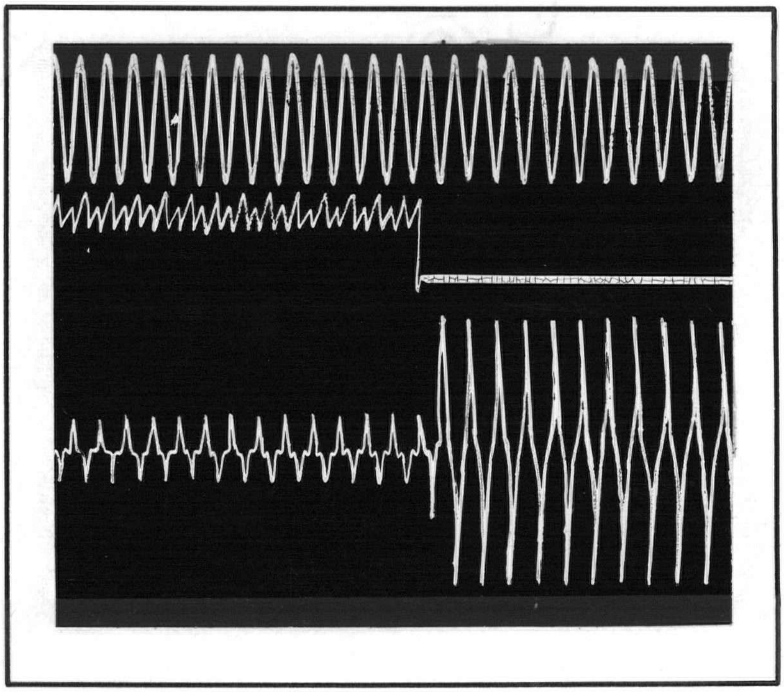


Fig. 31

Top Waveform 60 cycle sine wave-supply frequency

Middle Wavefrom control current waveform showing the second harmonics.

Bottom Waveform load current

Control current changed from 0 ma.to 13 ma.
Load current changed from .1 amp.to .47 amp.rms.
Load voltage changed from 50 volts to 120 volts rms.
Load resistance - 150 ohms.

Note the time lag in the bottom waveform which is seen to be one cycle of 60 cycle supply frequency.

VIII DISCUSSION

In magnetic amplifier design the maximum output within the linear range of the amplifier curve is an important factor,under certain conditions more so than the amplification factor.In determining this range the properties of the core material and the nature of the load are important.

More and better core materials are constantly being produced and the assumption that the BH curve can be approximated by three straight lines is becoming more and more correct.The knee of the curve is gradually being eliminated as Fig.16 shows and is almost absent from the material of the core in the magnetic amplifier used in the investigation.

The performance of the amplifier under different load parameters has been investigated and it has been found that the sensitivity was increased by the addition of a series or parallel condenser.This addition has a profound effect on the shape of the output wave causing a variation from a sine wave to a very peaky wave and also causing an output wave apparently twice the frequency of the supply voltage.The production of an output sine wave would be very useful if the output were to be fed to thermionic valves for the next stage of control.The production of a very peaky wave from the series circuit of Fig.11 might be of use where a large pulse of power was wanted without the use of thermionic valves.The range of the output impedance of the amplifier is in the range 25 to 2500 ohms and above that the output power is too low to be of any use.

The size of the magnetic amplifier used in this investigation and the comparable size of a transformer of the same rating shows that the magnetic amplifier is almost three times the size of the transformer.This would be detrimental if large power outputs were desired at 60 cycles.This can be overcome by raising the supply frequency where possible since the size of the magnetic amplifier is proportional to the the reciprocal of the frequency.

The investigation carried out has proven how difficult it is to analyze the performance of a magnetic amplifier.A special mathematical approach has still to be devised before any proper theoretical design principles can be found.The Germans in using trial and error methods of design have proven this to be true.Thus at present the only way to arrive at a real understanding of the properties of the magnetic amplifier is to use an oscillograph to trace the course of the currents.

IX CONCLUSION

In conclusion it may be interesting to reacpitulate some of the characteristics of practical magnetic amplifiers. This is done in order that potential users may judge for themselves their adaptability to various fields. These characteristics are:

1. They are very rugged, with no moving parts.
2. They have no filaments nor do they require a power pack .
3. They do not require time to heat up and are ready for action immediately the switch is closed.
4. There is no internal heat loss.
5. The input and output circuits can be completely isolated.
6. D.C. information can be converted to a.c. information and amplified also.
7. Signals at different voltage levels can be mixed.
8. Their size is three times that of a transformer of the same power rating.
9. Their size varies as the reciprocal of the supply frequency.
10. They take from 1 to 100 cycles of the supply frequency to respond completely to a change in control current.
11. Power gains of up to 10,000 can be obtained.
12. They have a low input impedance, .1 to 3000 ohms, and are suitable for amplification of thermocouple

output, photocell output etc.

13. Their output impedance is 25 to 2500 ohms thus they can be used where the load is to be a.c. or d.c. motors, heaters, recording apparatus etc.

14. They are in general low-impedance current-operated devices, and as such are complementary to electronic equipment and are not to be considered as rivals. Quite often it is found that a combination of a magnetic amplifier and valves is the best solution.

APPENDIX A

The circuits referred to this Appendix approximate to a series RC circuit across a sinusoidal input.

Let i be the current through the circuit, R and C the values of the resistance and capacitance respectively and E sin t the applied voltage. Then [4,5]

$$Ri + \frac{1}{C}\int i\,dt = E_m \sin \omega t$$

$$Rpi + \frac{i}{C} = E_m \frac{\omega p^2}{p^2+\omega^2}$$

$$i = \frac{E_m\omega}{R}\left[\frac{1}{p+\frac{1}{RC}}\right]\cos\omega t$$

$$= \frac{E_m\omega}{R}\,\frac{\left(\frac{1}{RC}\cos\omega t+\omega\sin\omega t-\frac{1}{RC}\,\varepsilon^{-\frac{t}{RC}}\right)}{\frac{1}{R^2C^2}+\omega^2}$$

APPENDIX B

THE USE OF FRÖLICH'S EQUATION

For any of the magnetic amplifier circuits when the variation of φ can be expressed as a simple function of the currents involved the relationship between current and time may be established mathematically as follows:

The basic relationship is:

$$Ri + \frac{d}{dt}(Li) = E_m \sin \omega t$$

$$\text{or} \quad Ri + L\frac{di}{dt} + i\frac{dL}{dt} = E_m \sin \omega t \text{ ------ } - \quad A(1)$$

however Frölich's equation, neglecting hysteresis effects, will give the relationship between flux and current to a reasonable degree of accuracy. Frölich's[2] equation is

$$\varphi = \frac{M}{1+ni}$$

where M and n are empirically determined constants for the particular magnetic circuit.

φ is the flux produced in the core by a d.c. current and an a.c. magnetizing current.

Therefore, since $L = N\varphi$, $L = \frac{NM}{1+ni}$

Substituting in equation $A1$ gives: $Ri + \frac{NM}{1+ni}\frac{di}{dt} - \frac{NMni}{(1+ni)^2}\frac{di}{dt} = E_m \sin \omega t$

$$\text{or} \quad \frac{di}{dt} = \frac{(1+ni)^2}{MN}\left[E_m \sin \omega t - Ri\right]$$

Let us now consider briefly the possibilities of finding analytical expressions for the results given in using Frölich's expression for the magnetization curve, the magnetic problem can be described by

$$\frac{di}{dt} = \frac{(1+ni)^2}{MN}\left[E_m \sin \omega t - Ri\right], \quad i(0) = 0.$$

This equation is not a standard type (Bernoulli, Riccati, etc.) and the solutions are therefore not likely to be elementary functions. However, the right side is continuous in i and t and possesses a constant partial derivative with respect to i; from Picard's[3] theorem, then, we know that a unique solution exists and can be found by the method of successive approximations. This method consists in choosing any zero-order approximation to the solution, say $i_0(t)$, then defining $i_1(t), i_2(t)$, ...through to

$$i_K(t) = \int_0^t \frac{(1+ni_{K-1})^2}{MN}\left[E_m \sin\omega t - Ri_{K-1}\right]dt.$$

Picard's theorem ensures the convergence of this series of functions to the solution. Expressions other than Frolich's can be treated in a similar manner. In any event the procedure would probably require the use of computing machines or numerical methods.

APPENDIX C

List of Symbols

a, b, c, d - coefficients of magnetization equation

B_o - amplitude of sinusoidal flux density

B_m - steady component of flux density

h_1 - excitation in core 1

H - magnetizing force in core ampere-turns/cm.

E_m - amplitude of the supply voltage

E - voltage drop across the magnetic amplifier

E_s - voltage across the magnetic amplifier required for saturation

E_{LOAD} - voltage drop across the load

E_d - control circuit voltage

i_a - load current

i_1 - current in the condenser branch

i_2 - current in the magnetic amplifier supply windings

i_d - current in the magnetic amplifier control windings

ω - $2\pi f$, where f is the supply frequency

a_1 - point on time base of supply voltage where core saturates

b_1 - point on time base of supply voltage where core desaturates

N_a - number of turns on each supply winding

N_d - number of turns on each control winding

N - total number of turns required to produce the flux φ if there were only one winding

φ - total flux in the core

φ_{ac} - flux produced by the a.c.magnetizing current

φ_d - flux produced by the control current

φ_1 - flux in core 1

φ_2 - flux in core 2

R_a - total resistance in the supply circuit

R_d - total resistance in the control circuit

C_a - capacity added in series or in parallel

X BIBLIOGRAPHY

1. Hauffe,G.,"Beitrag zur qualitativen Theorie gleichstromvormagnetisierter Eisenkern-Drosselspulen," Archiv fur Elektrotechnik,1939,33,page 41.
2. Kurtz,E.B.and Corcoran G.F.,"Introduction to Electric Transients",John Wiley and Sons,Inc.,New York,1935, page 230.
3. Reddick,H.W.and Miller,F.H.,"Advanced Mathematics For Engineers",John Wiley and Sons,New York,1946, page 165.
4. Coulthard,W.B.,"Transients in Electric Circuits",Sir Isaac Pitman and Sons Ltd.,London,1946.
5. Carter,G.W.,"The Simple Calculation of Electrical Transients",University Press,Cambridge,1944.