

THE MAGNETIC AMPLIFIER AND ITS
APPLICATION TO RADIO
FREQUENCY SIGNALS

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E. W. Steffen, Jr.

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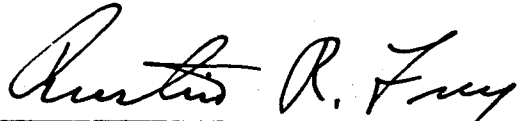
by

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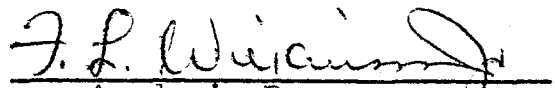
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PREFACE

This paper is presented as a partial fulfillment of the requirements for the degree of Master of Science to the United States Naval Postgraduate School. The laboratory work on this subject was performed at the Ward Leonard Electric Company in Mount Vernon, New York, during an eleven week period which commenced on January 5, 1948.

Grateful acknowledgment is given to Mr. Frank G. Logan, Mr. Warren Dornhoefer, Mr. Stephen Stienetz, and Mr. Erving Leedes, of the Ward Leonard Electric Company, for their guidance in this work.

The amplification of radio frequency signals with magnetic amplifiers is believed to be original work, hence, references to this subject are not available. All references listed in the bibliography concern magnetism and magnetic amplifiers for direct current or signals at power frequencies.

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LIST OF SYMBOLS AND ABBREVIATIONS

A	core area
A.C.	alternating current
B	flux density
B _e	flux density (initial value)
C	capacitance
cps	cycles per second
d	depth of flux penetration
D.C.	direct current
E _m	voltage (maximum value)
f	frequency
H	magnetizing force
i	current (instantaneous value)
l	core length
L	inductance
m	percent modulation
mmf	magnemotive force
N	number of turns
p	$2\pi\sqrt{\frac{\mu_0 f}{\rho}}$
P	power
Q	$\frac{\omega L}{R}$
R	resistance
t	time, or laminar thickness
W _e	eddy current loss (power)
W _h	hysteresis loss (power)

α	$[1 + \sin \gamma]^{\frac{1}{2}}$
γ	$\sin^{-1} 4\mu_0 \xi$
ξ	coefficient of hysteresis loss
$\bar{\Phi}$	power factor angle
μ	permeability
μ_0	initial permeability
Φ	magnetic flux
ω	angular frequency

INTRODUCTION

The magnetic amplifier, or saturable core reactor as they are often called, has been practically disregarded in this country since the time it was invented by E. F. W. Alexanderson in 1916. It has been used in a few instances, as in theater light dimming controls, but because of the lack of suitable magnetic materials, the magnetic amplifier has not been used to a great extent in the United States. Other countries, namely Germany and Sweden, have devoted much time and effort to magnetic amplifiers and have been able to develop magnetic materials, dry rectifiers and circuits which made the use of magnetic amplifiers feasible.

When the United States became aware of the potentialities of magnetic amplifiers, near the end of World War II, the United States Navy, various commercial firms, and schools embarked on a program of study and development which is continuing today. The magnetic amplifier is now in approximately the same stage of development that the vacuum tube was during the period immediately following World War I.

The magnetic amplifier has been utilized to date, predominately at frequencies from zero to 800 cps; although a successful amplifier for audio frequencies has been developed and demonstrated.*

* Ward Leonard Electric Company, Mount Vernon, New York.

The object of this paper, is to investigate the possible uses of magnetic amplifiers at radio frequencies, and to investigate the magnetic materials presently available. Radio frequencies will be defined here as frequencies exceeding 15 kilocycles per second.

It has been shown in this investigation that the magnetic amplifier can be adapted to amplifying signals at radio frequencies. The limit on the highest frequency that can be amplified is determined by the eddy current loss in the core material. As better core materials are developed, this limit on frequencies can be raised. The magnetic amplifier can be adapted for other uses, such as a reactance element to modulate the frequency of an oscillator or the amplitude of a radio frequency power source. The magnetic amplifier is superior to the vacuum tube amplifier in many instances where durability and dependability are prime requisites.

CHAPTER I

THE THEORY OF OPERATION

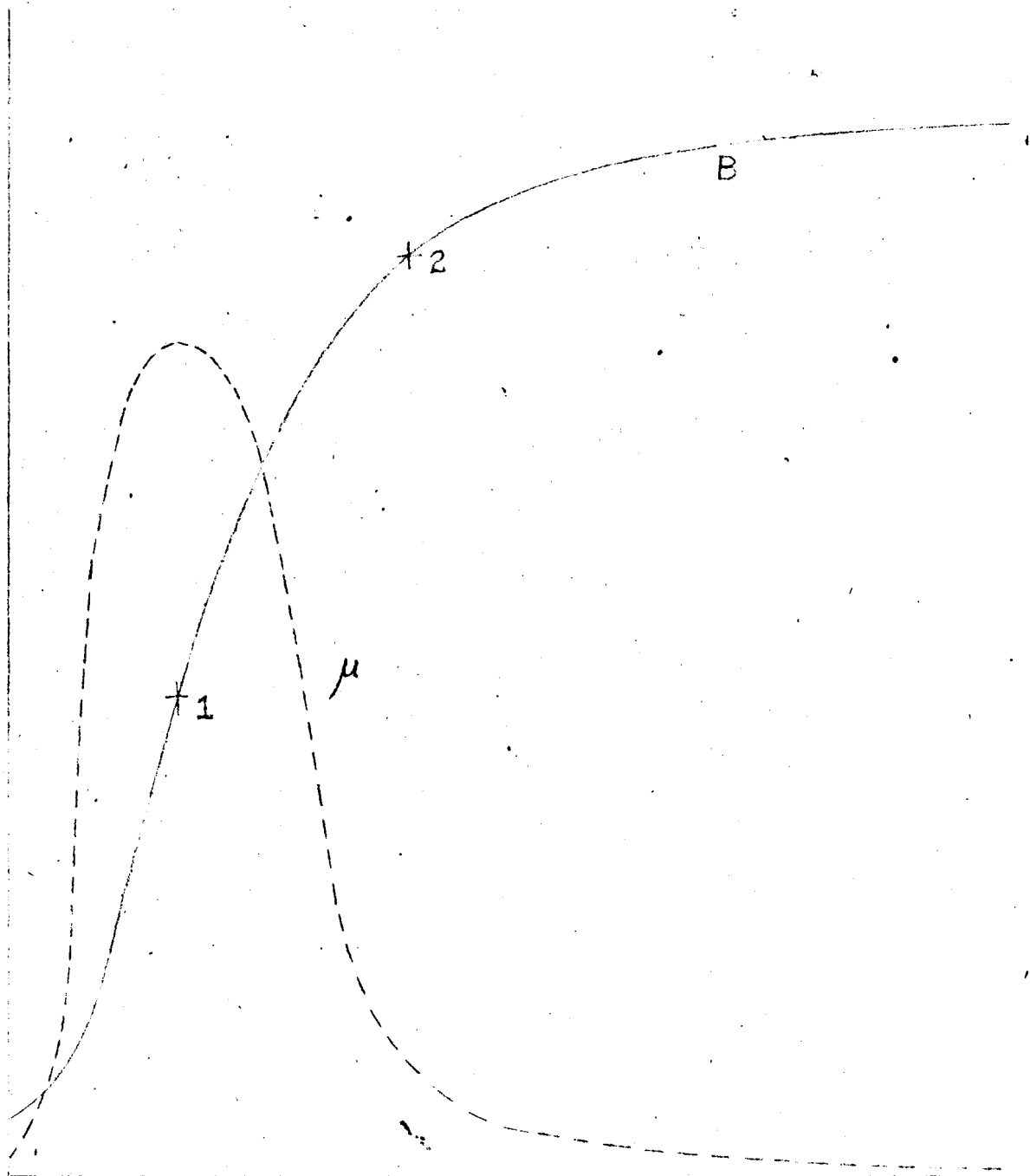
Any current carrying conductor possesses, to some degree, the property of inductance. When the magnetic field of this conductor is induced in a magnetic material, the inductance is changed, depending upon the geometry of the magnetic core and its permeability. Using cgs units, the inductance L , of a core of mean length ℓ , area A , with N turns of conductor, and of permeability μ , is:

$$L = \frac{4\pi N^2 A \mu}{\ell} \times 10^{-9} \quad \text{heneries} \quad (1)$$

The permeability, μ , which is defined as the flux density divided by the magnetization force, (B/H) , is the only variable factor for any given core in Equation (1). Hence, if the permeability of the magnetic material can be made to vary in a desired manner, the reactance and the current through the coil or reactor is a function only of μ .

In Fig. 1 the flux density of a typical core is plotted as a function of the magnetization force, the common D.C. magnetization curve. Also plotted is the permeability of the core as a function of magnetizing force H . It will be seen that the permeability starts at some definite value, at H equal to zero. The permeability increases to a maximum

Flux Density, B, or Permeability, μ .



Field Strength, H.

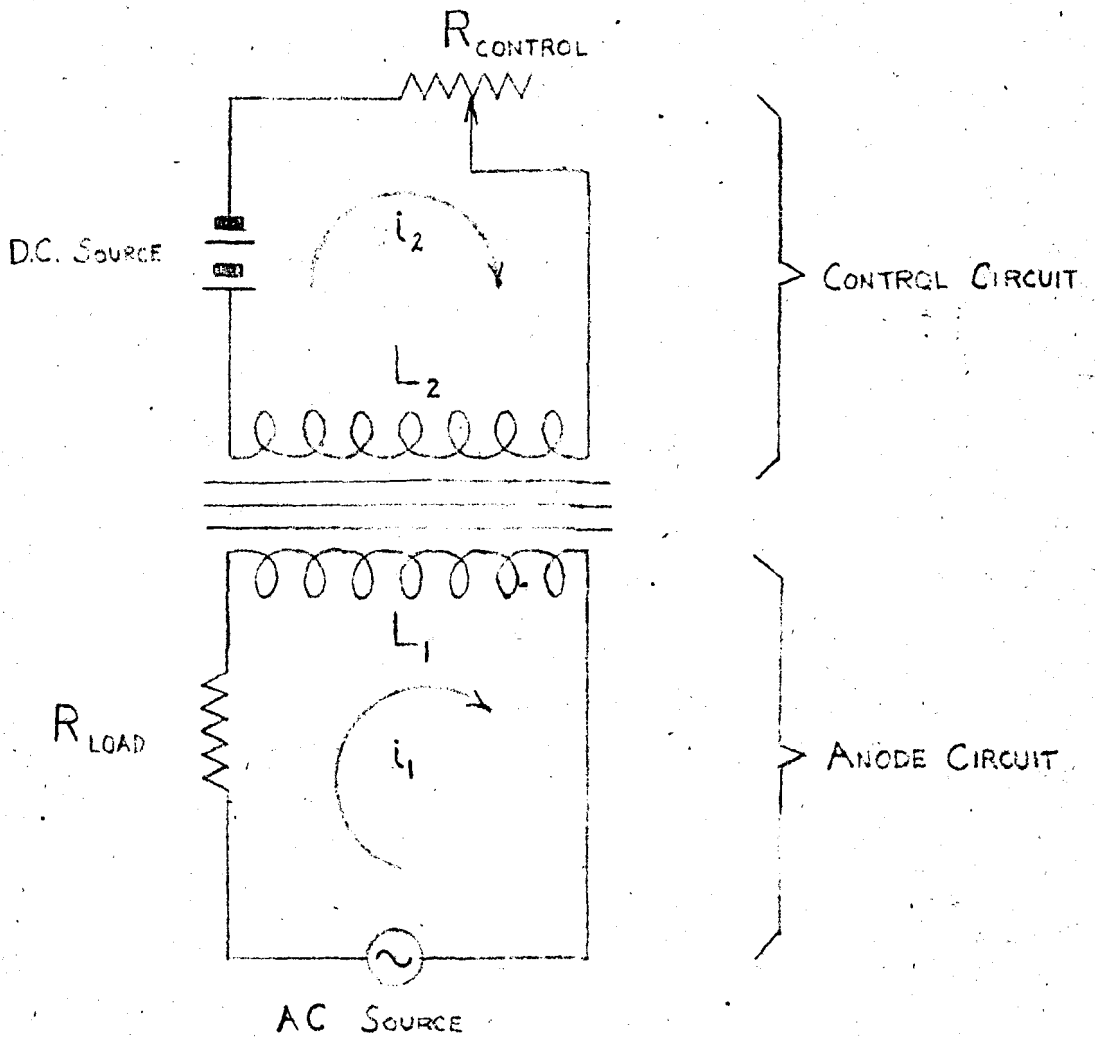
VARIATION OF FLUX DENSITY AND PERMEABILITY WITH
MAGNETIZING FORCE

FIGURE 1.

value , and at the knee of the magnetization curve, decreases rapidly as H increases, approaching zero value asymptotically.

If the permeability of a reactor core can be made to vary in one of the linear regions of this curve, then the coil reactance and hence the current through the reactor, will vary linearly. One of the methods used to obtain this linear change in permeability is to operate on the knee of the magnetization curves so that the change of μ is a linear function of the magnetizing force, H.

Let us consider a simple magnetic amplifier, such as that illustrated in Fig. 2. Neglecting the upper half of Fig. 2 the control circuit, for a moment, the current, i_1 will be limited by the values of R_{load} and the reactance of L_1 , where L_1 will be given by Equation (1). Now however, if direct current is allowed to flow in the upper circuit or control circuit of Fig. 2, the permeability of the magnetic core material will be changed, since i_2 will cause the operating point, 1, (see Fig. 1) to shift to point 2. Thus, this change in permeability of the core material has caused the reactance of L_1 to decrease, allowing more current to flow through the load resistance. In this manner, by the control of the current i_2 , the current flowing in the load circuit, and hence the voltage developed across R_{load} , can be controlled.



BASIC MAGNETIC AMPLIFIER CIRCUIT

FIGURE 2.

Referring again to Fig. 1, we may choose the operating limits of the magnetization force, H , such that the corresponding change of permeability is linear, or nearly so.

The successful operation of this circuit as an amplifier then depends on the sensitivity of control, the linear change of permeability with magnetizing force, and the gain obtainable per stage. At low frequencies (below 800 cps) various magnetic amplifiers have been built which respond to a fraction of a microvolt, give power gains in the order of 100,000, and have very low harmonic content. The harmonic content, however, will depend upon the type of operation.

VerPlank and Fishman (24) have analyzed the circuit shown in Fig. 2. In their analysis, the resistances of the anode and control circuits, the flux leakage, and the core losses were neglected, and a uniform flux density was assumed. At low frequencies and with proper core design, these assumptions are valid except for neglecting resistances. If the values of R_{load} and $R_{control}$ and the winding resistances are small compared to the winding reactances, then these solutions will be approximately correct.

Considering the voltages in the anode circuit:

$$E_m \cos \omega t = N_1 \frac{d\phi}{dt} \quad (2)$$

where $E_m \cos \omega t$ is the applied voltage, N_1 is the number of turns in the control winding, and $\frac{d\phi}{dt}$ is the time rate of change of the total flux linking the load circuit winding.

The solution of Equation (2) is:

$$B = \frac{\phi}{A} = B_m \sin \omega t + B_0 \quad (3)$$

where B is the flux density, A the core area, and B_0 the initial flux density. The maximum flux density B_m is defined as:

$$B_m = \frac{\phi_m}{A} = \frac{E_m}{2N_1 \omega A} \quad (4)$$

The relationship between the control current i_2 , and the load current i_1 , can be derived from a relationship of mmf's. These solutions depart considerably from actual wave forms obtained by oscillographic methods, due mainly to the effect of resistances present in both the load and control circuits.

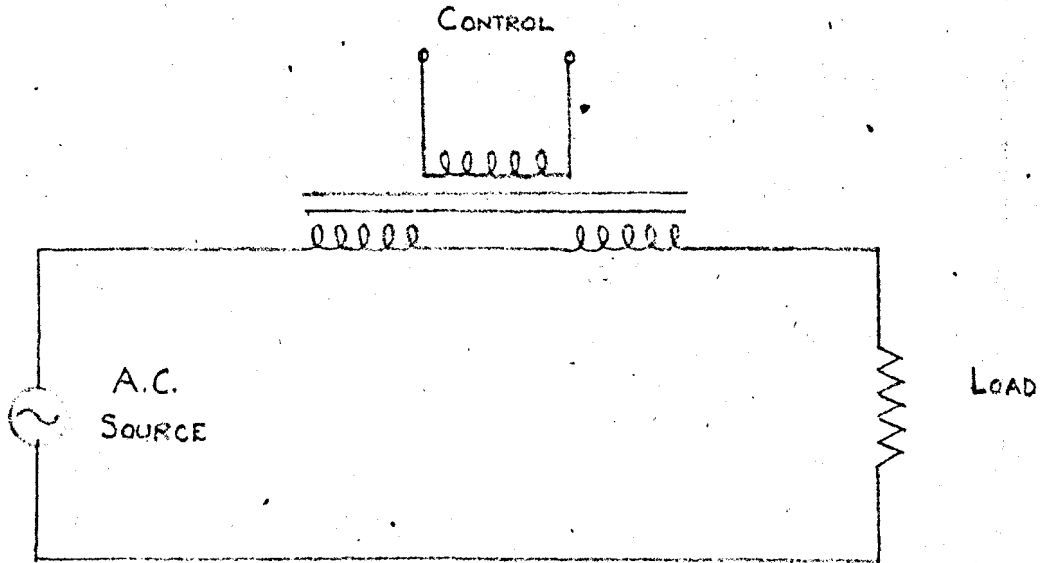
To avoid the use of vacuum or gas tubes, dry rectifiers are used in most magnetic amplifiers to obtain

direct currents when needed. The characteristics of these rectifiers are very important in the functioning of a magnetic amplifier, since a small amount of back current through a rectifier may act as an effective control current and can cause a marked effect on the amplifier response.

Greater sensitivity to control current may be obtained by the use of feedback similar to that in vacuum tube amplifiers. A more complete discussion of feedback will be presented later.

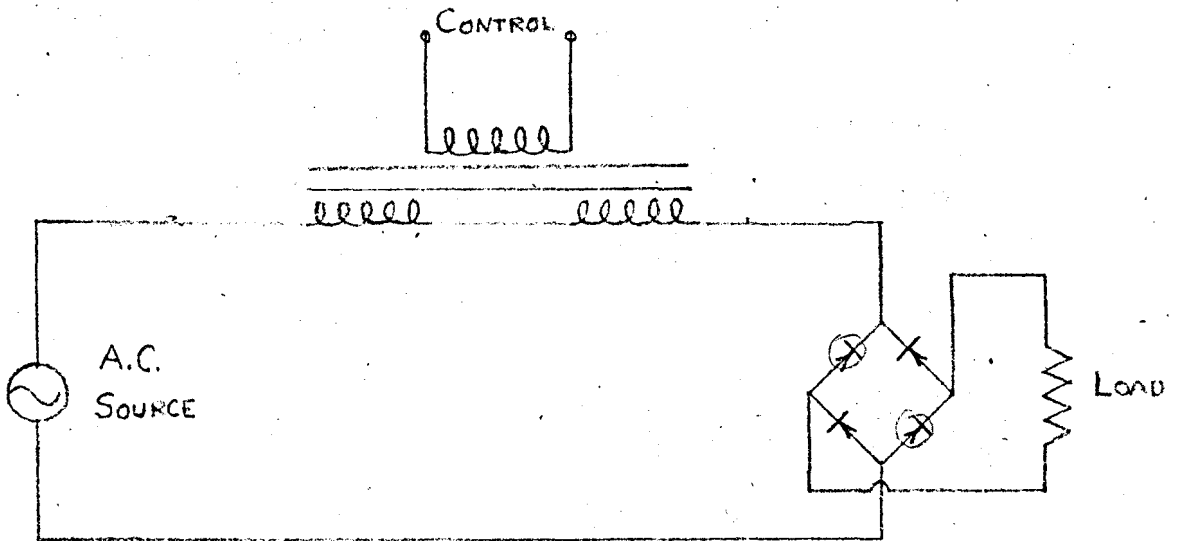
Another important factor which cannot be overlooked is the transient response of an amplifier. Since the load circuit consists of resistance and inductance, there is an exponential delay in the time between the applied signal voltage and the current through the load resistor. In nearly all circuits this time constant is an important limitation on the speed of response. The use to which a magnetic amplifier is to be put will usually determine the time constant characteristics. By proper choice of resistance, inductance, and capacitance, this effect may be minimized.

Magnetic amplifiers may be used to control either alternating or direct current through a load resistor as shown in Fig. 3. In Fig. 3 (a) the basic A.C. amplifier controls the alternating current produced by the power



CONTROL OF A.C. CURRENT THROUGH LOAD

(a)



CONTROL OF D.C. CURRENT THROUGH LOAD

(b)

FIGURE 3.

source. In Fig. 3 (b) a bridge rectifier has been added to make the circuit a D.C. amplifier. This circuit lends itself to the application of actuating relays for on-off control and in such applications as computers. Smoothing elements may be added across the output if desired.

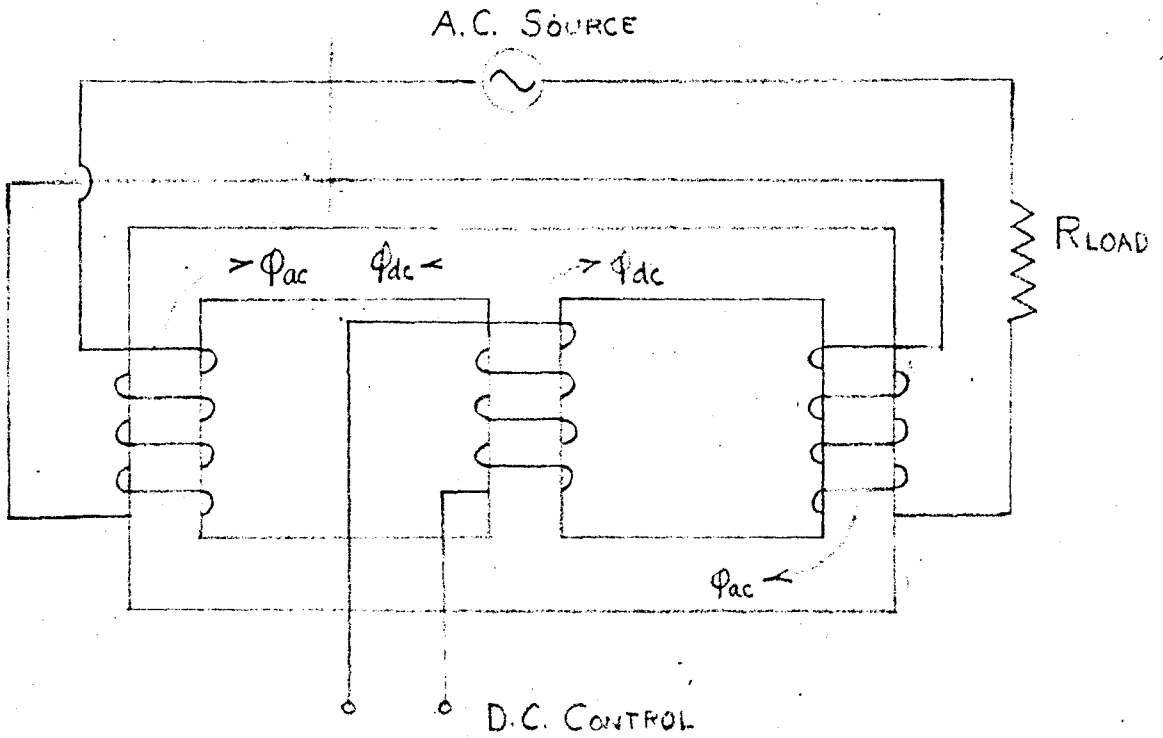
Bias flux is often added to a reactor core to control the quiescent operating point about which the control flux may vary. This flux is obtained by adding another winding to the core and passing direct current through it. This D.C. bias current is normally obtained from the A.C. power source through a bridge rectifier. The bias current is set to place the core in a partially saturated condition such that the amplifier may act as either a Class A, Class B, or Class C amplifier.

Fig. 2 shows the most simple type of magnetic amplifier; it possesses some inherent objections, however. First, unless the D.C. current source in the control circuit has infinite internal impedance, the A. C. flux from the load circuit will set up an alternating current in the control circuit. This A.C. current in the control circuit has the effect of shorting out some of the turns on the load circuit winding L_1 . Hence, when the value of R_{control} is changed, the voltage appearing across R_{load} will be a function of the A.C. and D.C. components of i_2 . A more desirable arrangement would be for a change of load

current to be a function only of the D.C. control current. This effect is accomplished in several ways. One method is to use a core configuration such that none of the A.C. flux passes through the part of the core on which the D.C. control windings are placed.

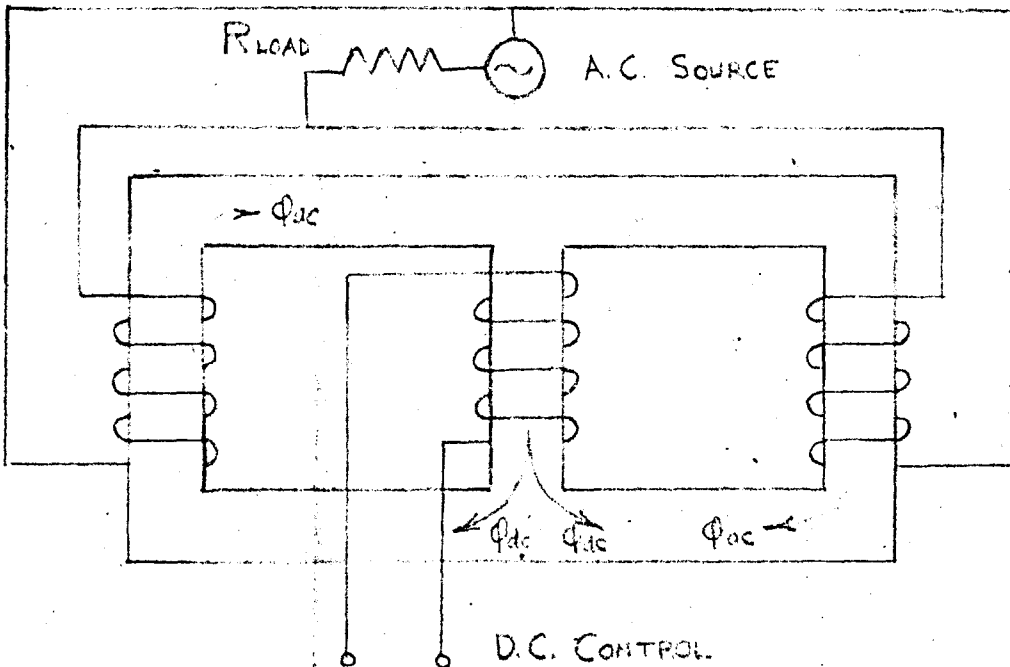
Such an arrangement is shown in Fig. 4. In this reactor, the alternating current coils are in series, and so wound or connected that their instantaneous mmf's are opposed in the center leg and thus should induce no A.C. voltage in the D.C. control winding. When direct current flows in the control winding however, D.C. flux exists in the entire core which is then subjected to a degree of saturation which is a function of the ampere turns of the control coil and the characteristics of the core material. The degree of saturation affects the reactance of the A.C. windings and this may be adjusted from a maximum with no control current flowing to some practical minimum value determined chiefly by the leakage reactance of the structure and the heating of the control windings.

The number of control D.C. ampere turns, and hence the control power, required to saturate the core structure and thus reduce the reactance to a minimum, can be predicted rather closely for any given set of conditions, provided the core characteristics are known. Again considering Fig. 4, it is seen that the control mmf opposes the mmf of one A.C. winding at any instant. This means that in



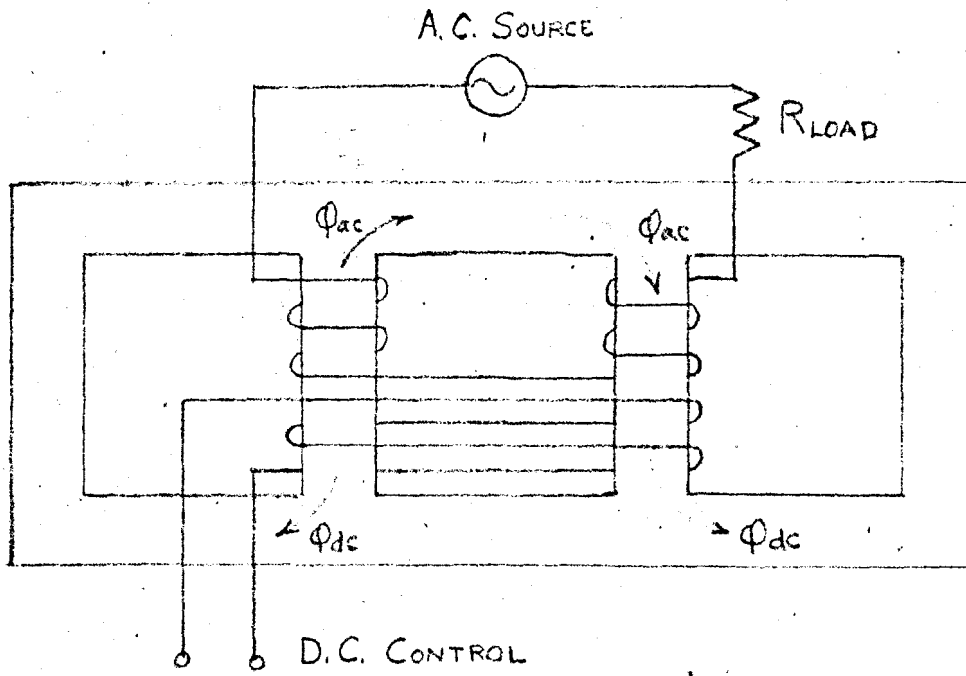
SERIES FED AMPLIFIER

FIGURE 4.

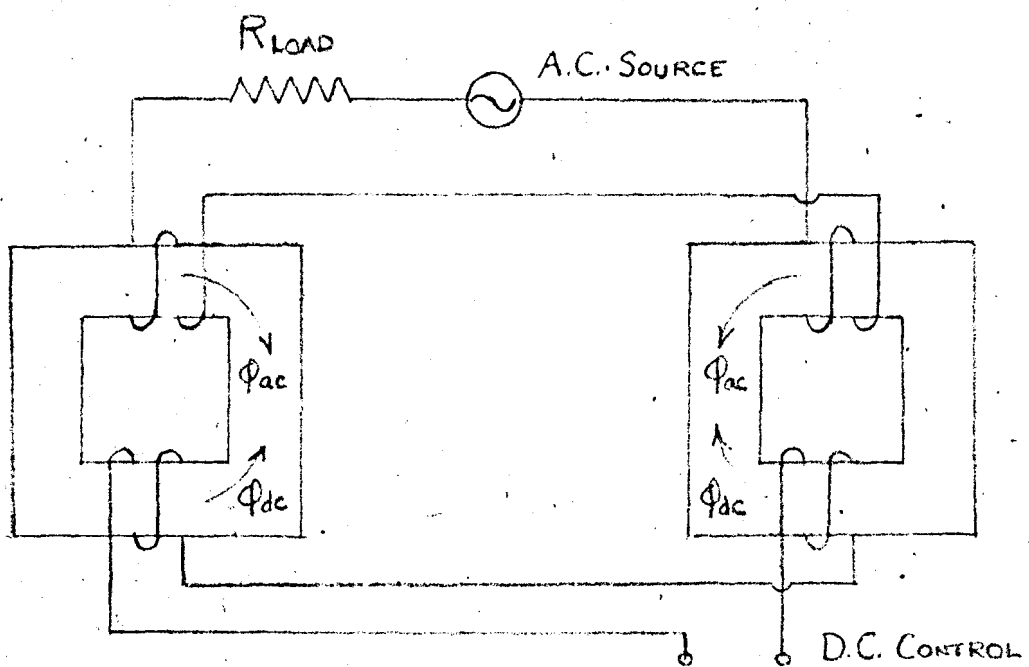


PARALLEL FED AMPLIFIER

FIGURE 5.



FOUR LEGGED AMPLIFIER
FIGURE 6.



TWO CORES WITH A.C. VOLTAGE INDUCED
IN D.C. WINDING CANCELING
FIGURE 7.

one leg, for complete saturation enough control ampere turns must be provided to cancel out the mmf produced by the peak A.C. current and also to saturate the core in this portion of the structure to an extent which will bring it up far enough on its saturation curve to reduce the coil reactance to the desired minimum. In practice, the number of ampere turns per unit length must be limited due to excessive flux leakage.

Alternating current may also be eliminated from the control windings by using two cores such as the circuit shown in Fig. 7. In this circuit, two separate cores are used in place of a single multi-legged core. While the A.C. flux will link both of the control windings, if the windings on each core are identical the control windings may be connected so that the resulting induced A.C. voltages are bucking and will cancel out. The A. C. or anode windings may be connected in parallel if so desired. An objection to this general configuration is that although the net induced voltage across the control terminals is zero, the induced voltage across either one of them may become dangerously high. This circuit is mainly used in small saturable core reactors where the cores are of high permeability materials. In such a case where control is easily obtained, the number of turns on the control windings may be small.

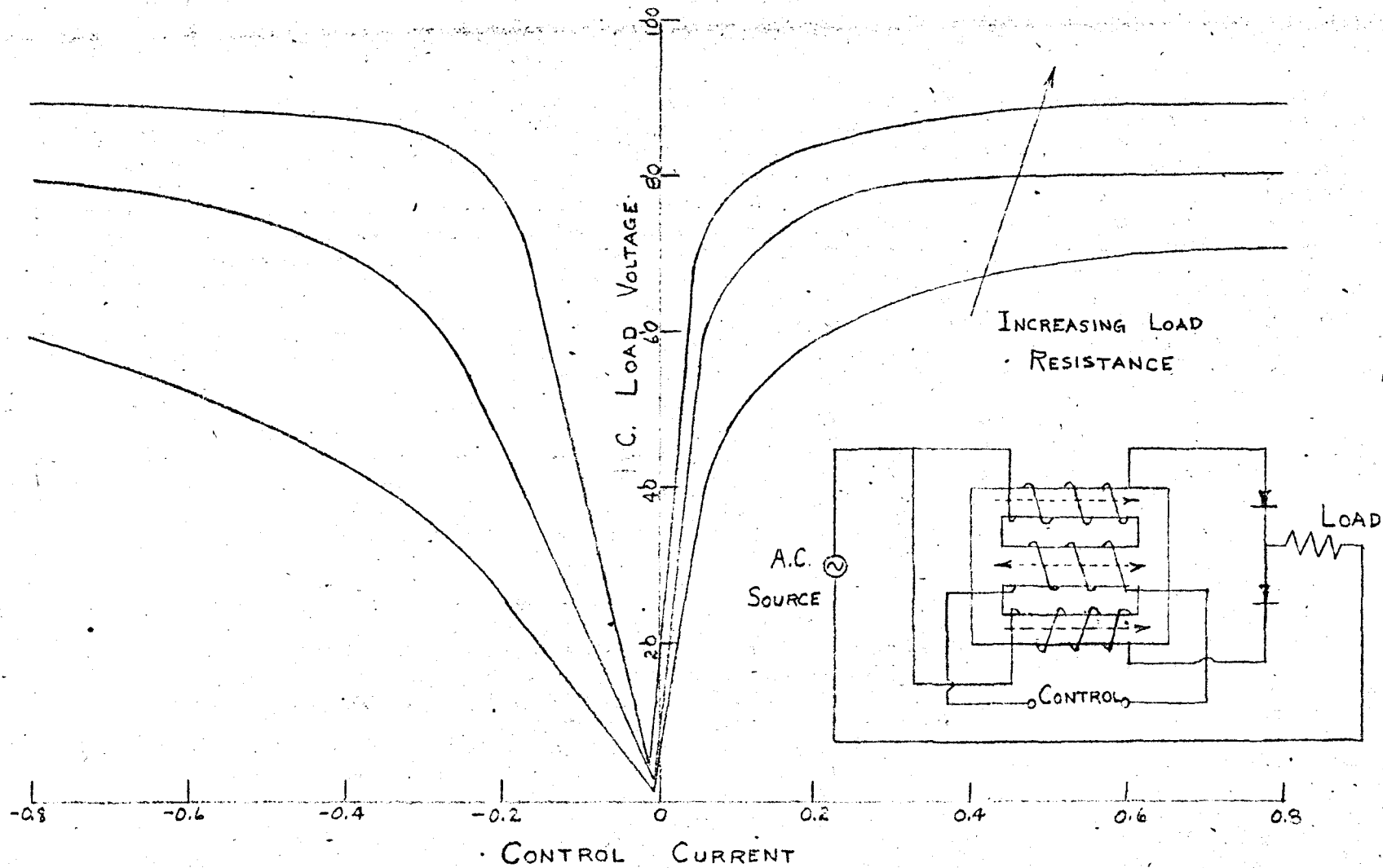
CHAPTER II

CIRCUITS

1. Magnetic Amplifiers for Low Frequencies.

Several important circuits are used in magnetic amplifiers operating at low frequencies. One of these circuits is shown in Fig. 8. F. G. Logan (20) introduces with this circuit, a form of self-saturation. The windings on the outside legs carry half wave unidirectional current. With no control current, each half of the core will be left with residual flux after each conducting half cycle and the whole reactor will possess less inductance than before the core halves were biased by the half wave D.C. load current. The effect of this self-saturation is to increase the load current without external power. This type of operation is essentially feedback without the extra windings necessary. For similar circuits conditions, the degree of self-saturation is determined largely by the core material. The output current resulting from this action may be as high as fifty percent of maximum output with high permeability core materials. Maximum output is achieved by additional saturation of the core caused by the control current.

In this type reactor it may be seen that the control mmf is alternately aiding and opposing the mmf of



CONTROL CHARACTERISTICS
 SELF SATURATING TYPE — MAGNETIC AMPLIFIER

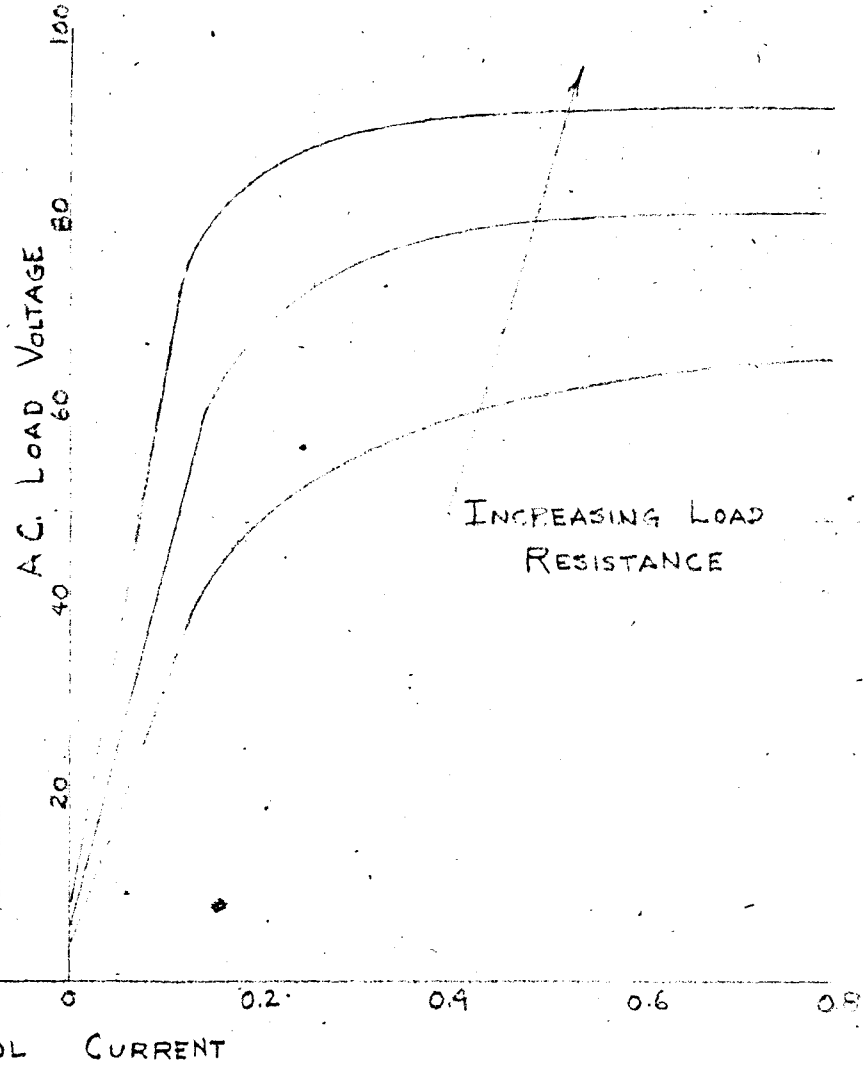
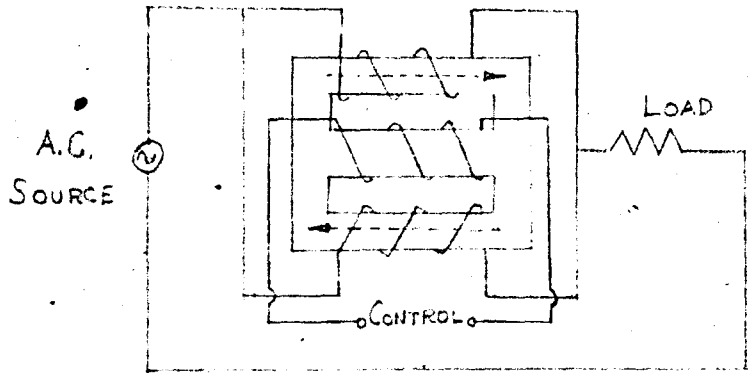
FIGURE 8.

the half wave power supply current passing through the rectifiers. The control mmf required to produce a certain degree of saturation is equal to the total mmf required minus that contributed by the load current. Also plotted in Fig. 8 are the control characteristics of this type amplifier. These control characteristics show the voltage across various values of load resistance as a function of the control current. The effect of removing self-saturation on the control characteristics of this type amplifier is shown in Fig. 9 with its associated circuit. It will be noted that self-saturation allows the amplifier to be responsive to control current flowing in either a negative or positive direction. The reactor shown in Fig. 9 may be made to respond to a reversible or A.C. control current if a separate bias winding is added to the center leg of the reactor. The D.C. bias current may then be adjusted to allow a certain quiescent voltage to appear across the load. Superimposing the control flux in the core will then vary the voltage across the load. From the control characteristics in Fig. 9 it can be seen that by biasing the reactor near the knee, on the upper part of the curve, the amplifier may be operated as a Class B or Class C amplifier. Biasing the reactor in the lower linear portion of the curve will give Class A operation. Distortionless amplification will result providing this region of operation is perfectly linear.

Although the amplifiers described here employed the three legged type reactor shown in Fig. 5, other configurations such as that shown in Fig. 6 may also be employed, depending on the particular use of the amplifier. With higher permeability core materials, the control characteristics shown in Figs. 8 and 9 will be much steeper, showing much better sensitivity. Magnetic materials which have been annealed under strong magnetic fields may have a very sharp knee on both the magnetization curve and on the control characteristics. For Class B or Class C amplification this sharp knee is advantageous in that it gives a sharp cut-off, similar to the sharp cut-off which may be obtained with some types of gas or vacuum tubes.

Although these amplifiers have been described as having D.C. control, with proper biasing they will operate well for amplifying A.C. signals. The difference being the transient response of the circuit used. The time constant of a circuit can be improved by increasing the ratio of resistance to inductance at the sacrifice of power gain. At higher frequencies the addition of capacitance will improve the circuit time constant further. Response time of magnetic amplifiers can also be shortened by incorporating first differential positive feedback.

Because the magnetic amplifier is a power operated device, the impedance of the signal or control source should match that of the control winding for optimum



CONTROL CHARACTERISTICS
NON-SATURATING TYPE - MAGNETIC AMPLIFIER

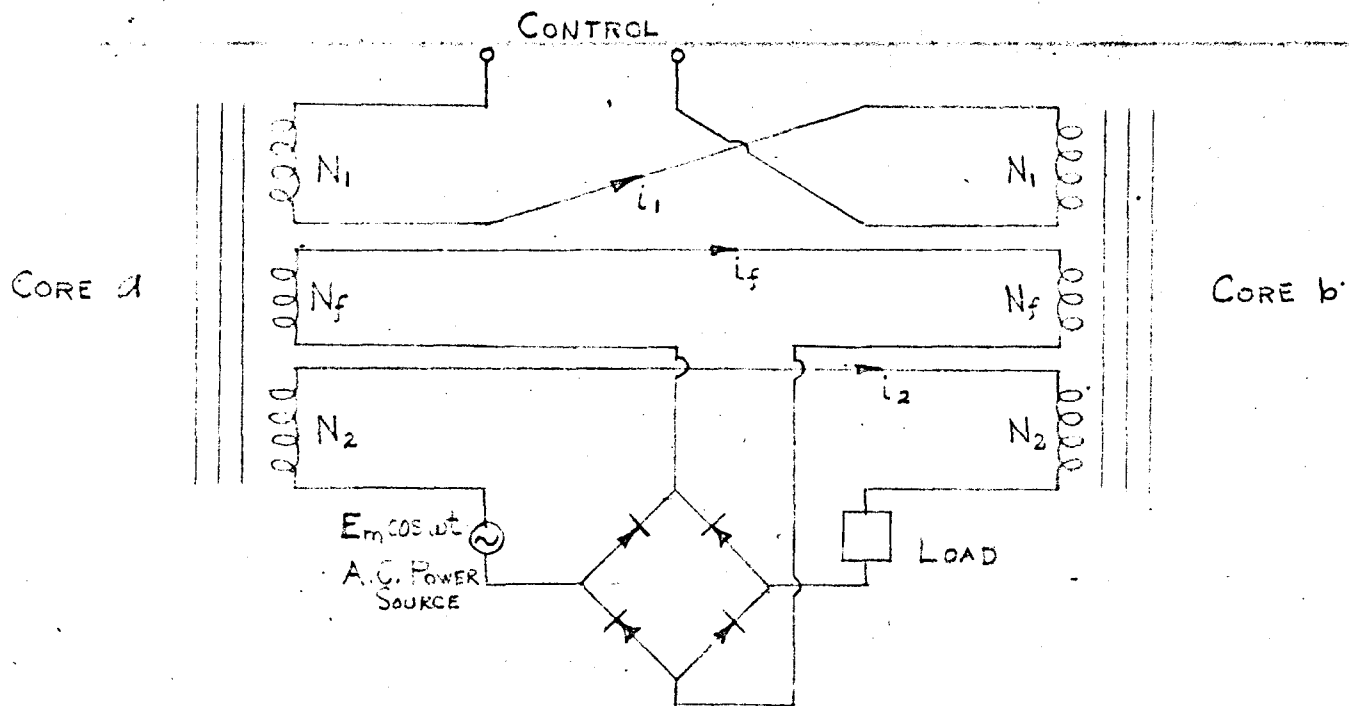
FIGURE 9.

performance. For the same reasons, the load should be matched to the amplifiers. This matching can usually be best accomplished by building transformers as part of the circuit.

One advantage of the magnetic amplifier is the complete isolation of the input and output terminals. The greatest amplification can be obtained with resistive loads. Two amplifiers may be connected in push-pull to give zero load current with zero input or control signal.

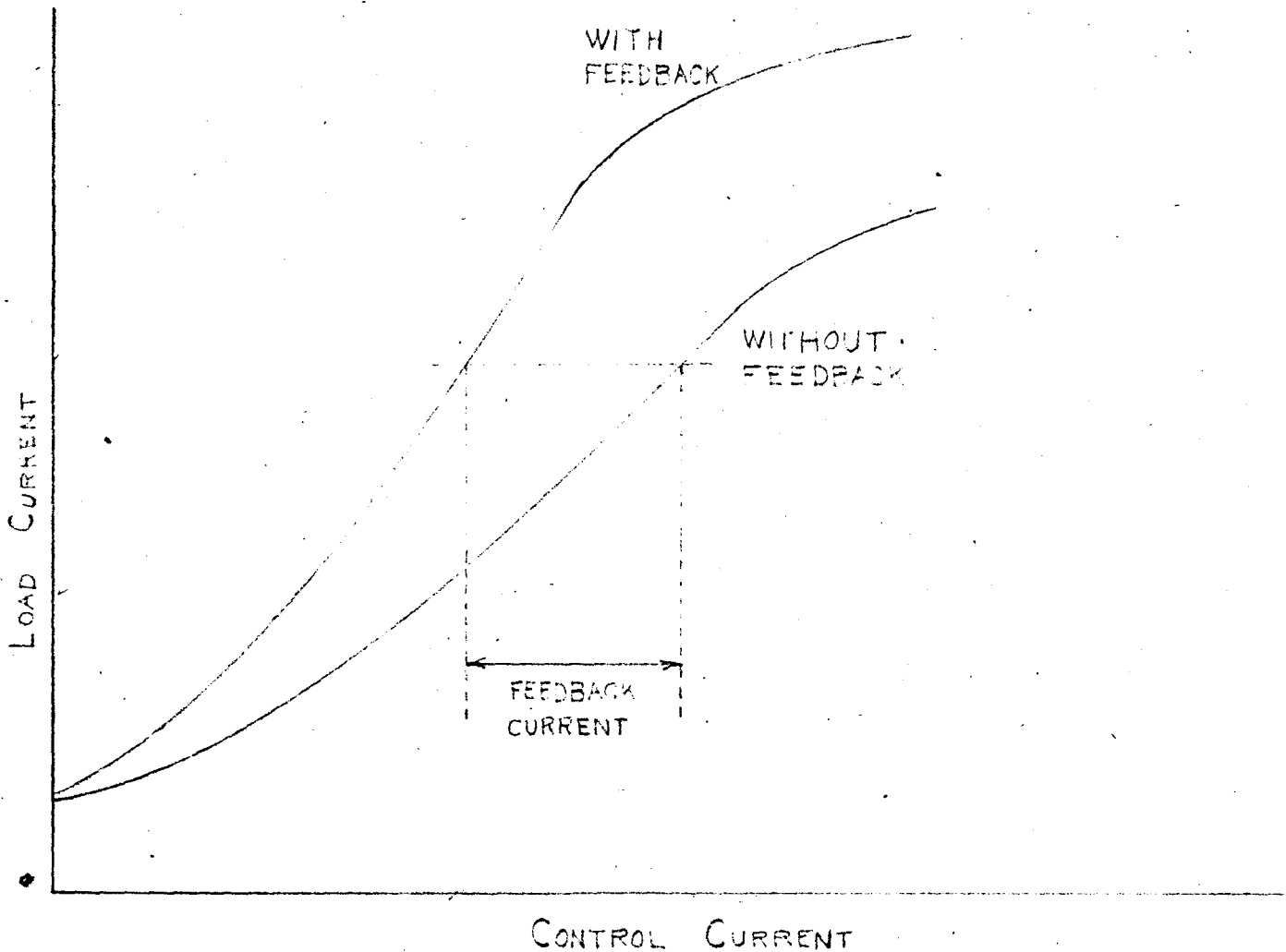
2. Feedback.

A German scientist, Dr. Theodore Buchold (6) (16), doing basic research on magnetic amplifiers has demonstrated that by the use of feedback, the necessary control power can be reduced. Figure 10 shows a basic circuit in which the primary, secondary, and tertiary windings represent the anode, control, and feedback circuits. Referring to Fig. 11, the action of feedback is shown on the control characteristic curves. One curve shows the load current as a function of control current for a magnetic amplifier such as shown in Fig. 2. If feedback is added, such as with the circuit shown in Fig. 10, better control is obtained, that is, a greater apparent variation in load current for the same control current. While the same amount of control current is required for both circuits, part of this control current is furnished by the feedback



SIMPLE FEEDBACK CIRCUIT FOR SERIES CONNECTED REACTOR USING SEPARATE FEEDBACK WINDINGS

FIGURE 10.



CONTROL CHARACTERISTICS OF MAGNETIC AMPLIFIER WITH FEEDBACK

FIGURE 11.

windings. Care must be taken with an amplifier utilizing feedback since oscillations will occur if a signal is fed back which satisfies the Barkhausen criterion for oscillation.

VerPlank and Fishman (24) have analyzed the circuit shown in Fig. 10. In their analysis they neglected all resistances, assumed the flux density was uniform. Neglected flux leakage, and assumed identical characteristics for each of the four rectifiers. With proper precautions, all these assumptions are valid except that of neglecting the resistances. The effect of resistance will modify somewhat their results. The cycle of operation is divided into three periods and the voltage equations written for each period.

During the interval when i_2 is positive, a period exists when only two rectifiers are in operation. Then:

$$\begin{aligned}
 E_m \cos \omega t &= N_z A \frac{dB_a}{dt} + N_z A \frac{dB_b}{dt} - N_f A \frac{dB_b}{dt} + N_f A \frac{dB_a}{dt} \\
 &= (N_z + N_f) A \frac{dB_a}{dt} + (N_z - N_f) A \frac{dB_b}{dt} \quad (5)
 \end{aligned}$$

where $E_m \cos \omega t$ is the applied voltage; A , the cross-sectional area; N_z , the number of turns per load winding; N_f , the feedback turns; B_a , the flux density in core "a";

and B_b , the flux density in core "b".

During the interval when i_2 is negative, the two rectifiers which were not conducting during period one, will be conducting. The voltage equation may be written as:

$$E_m \cos \omega t = N_z A \frac{dB_a}{dt} + N_z A \frac{dB_b}{dt} + N_f A \frac{dB_b}{dt} - N_f A \frac{dB_a}{dt}$$

$$= (N_z - N_f) A \frac{dB_a}{dt} + (N_z + N_f) A \frac{dB_b}{dt} \quad (6)$$

During the third period when all arms of the rectifier bridge are conducting, that is, if $\frac{i_f}{2} - \left| \frac{i_2}{2} \right| > 0$ or if $i_f > |i_2|$, then:

$$0 = -N_f A \frac{dB_b}{dt} + N_f A \frac{dB_a}{dt}$$

$$= \frac{d}{dt} (B_a - B_b) \quad (7)$$

Solving these equations gives:

$$B_a = B_m \sin \omega t + B_1 \quad (8a)$$

and

$$B_b = B_m \sin \omega t - B_1 \quad (8b)$$

where:

$$B_m = \frac{E_m \times 10^8}{2\omega N_2 A} \quad \text{gauss}$$

and B_1 , is the average flux of path "a". From these equations the flux density or voltage across any circuit element may be calculated at any instant. It must be remembered that the effect of resistance may effectively alter these equations in some instances.

CHAPTER III

THE EFFECT OF RADIO FREQUENCY FIELDS ON CORE CHARACTERISTICS

Before proceeding with a discussion of the results of the application of radio frequency signals, it will be necessary to discuss some of the theoretical treatments of ferro-magnetic materials. It is well known that neglecting hysteresis loss, the apparent permeability of laminae may be expressed in absolute units by:

$$\mu = \frac{\mu_1}{\sqrt{tp}} \left[\frac{\cosh tp - \cos tp}{\cosh tp + \cos tp} \right]^{\frac{1}{2}} \quad (9)$$

where:

$$\rho = 2\pi \sqrt{\frac{\mu_1 f}{\rho}} \quad (11)$$

μ_1 , is the true of D.C. permeability, t the thickness of the laminae, ρ the specific resistivity, and f the frequency.

When tp reaches the value of 3.0, the value of the quantity in brackets of Equation (9) has reached a value differing from unity by only ten per cent. The limiting value of μ then becomes:

$$\mu = \frac{\mu_1^{\frac{1}{2}} \rho^{\frac{1}{2}}}{2\pi t} \cdot \frac{1}{f^{\frac{1}{2}}} \quad (10)$$

Dannatt (8) expresses the associated eddy current loss, the reaction of which causes the apparent loss of permeability, as:

$$W_e = \frac{\rho \rho H_0^2}{16\pi t} \left[\frac{\sinh tp - \sin tp}{\cosh tp + \cos tp} \right] \quad (12)$$

which approaches the limiting value of:

$$W_e = \frac{\mu_1^{\frac{1}{2}} f^{\frac{1}{2}} \rho^{\frac{1}{2}} H_0^2}{8\pi t \times 10^7} = \frac{\pi t f^{\frac{3}{2}} B_a^2}{4\mu_1^{\frac{1}{2}} \rho^{\frac{1}{2}} \times 10^7} \quad (13)$$

where B_a is the apparent or measured flux density, maximum instantaneous value.

From Equations (9) and (12) the power factor may be found to be:

$$\cos \Phi = \frac{\frac{\sqrt{2}}{2} \left[\frac{\sinh tp - \sin tp}{\cosh tp + \cos tp} \right]}{\left[\frac{\cosh tp - \cos tp}{\cosh tp + \cos tp} \right]^{\frac{1}{2}}} \quad (13)$$

The limiting value of $\cos \Phi$ at large values of tp is:

$$\cos \Phi = \frac{\sqrt{2}}{2} = 0.707 \quad (14)$$

or the angle between the mmf and the effective induction is $\pi/4$.

Introducing the effect of hysteresis loss, to be added to the eddy current loss, gives:

$$W_e + W_h = \frac{\omega^2 \alpha}{4\rho p} \left[\frac{\alpha \sinh t p \alpha - \beta \sin t p \beta}{\cosh t p \alpha - \cos t p \beta} \right] B_a^2 10^{-7} \quad (15)$$

where,

$$\alpha = [1 + \sin \mathcal{J}]^{\frac{1}{2}} \quad \text{and} \quad \beta = [1 - \sin \mathcal{J}]^{\frac{1}{2}}$$

and

$$\sin \mathcal{J} = 4\mu_0 \mathcal{J}$$

where \mathcal{J} is the coefficient of hysteresis loss. As tp approaches a large value (high frequency), Equation (15) approaches:

$$W_c + W_h = \frac{\pi \alpha t f^{3/2} B_a^2}{2 \mu_i^{1/2} \rho^{1/2} 10^7} \quad (16)$$

and the expression for the power factor is modified to the value of:

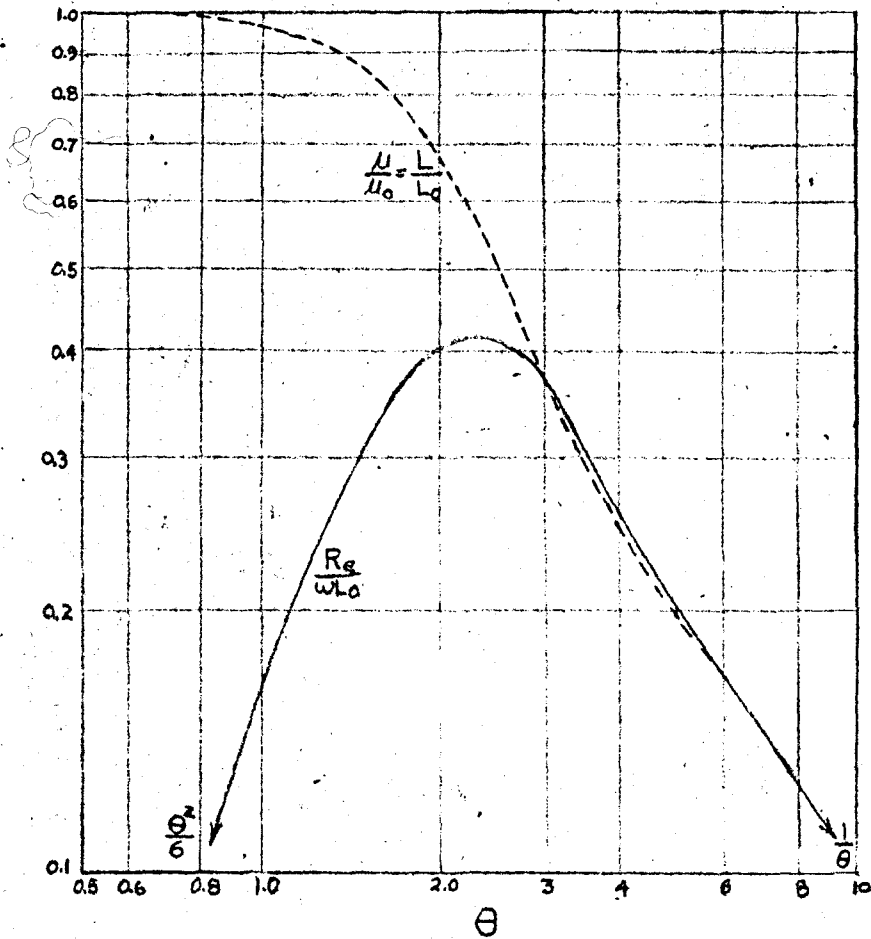
$$\cos \Phi = 0.707 \alpha \quad (17)$$

In Fig. 12 the ratio of apparent permeability, μ and $\frac{1}{Q}$ or $\frac{R}{\omega L}$ are plotted as functions of θ , where:

$$\theta = \rho t = 2\pi t \sqrt{\frac{\mu_i f}{\rho}} \quad (18)$$

Fig. 13 shows Equation (9) plotted as a function of frequency for Supermalloy of thicknesses of 0.001, 0.002, and 0.003 inches.

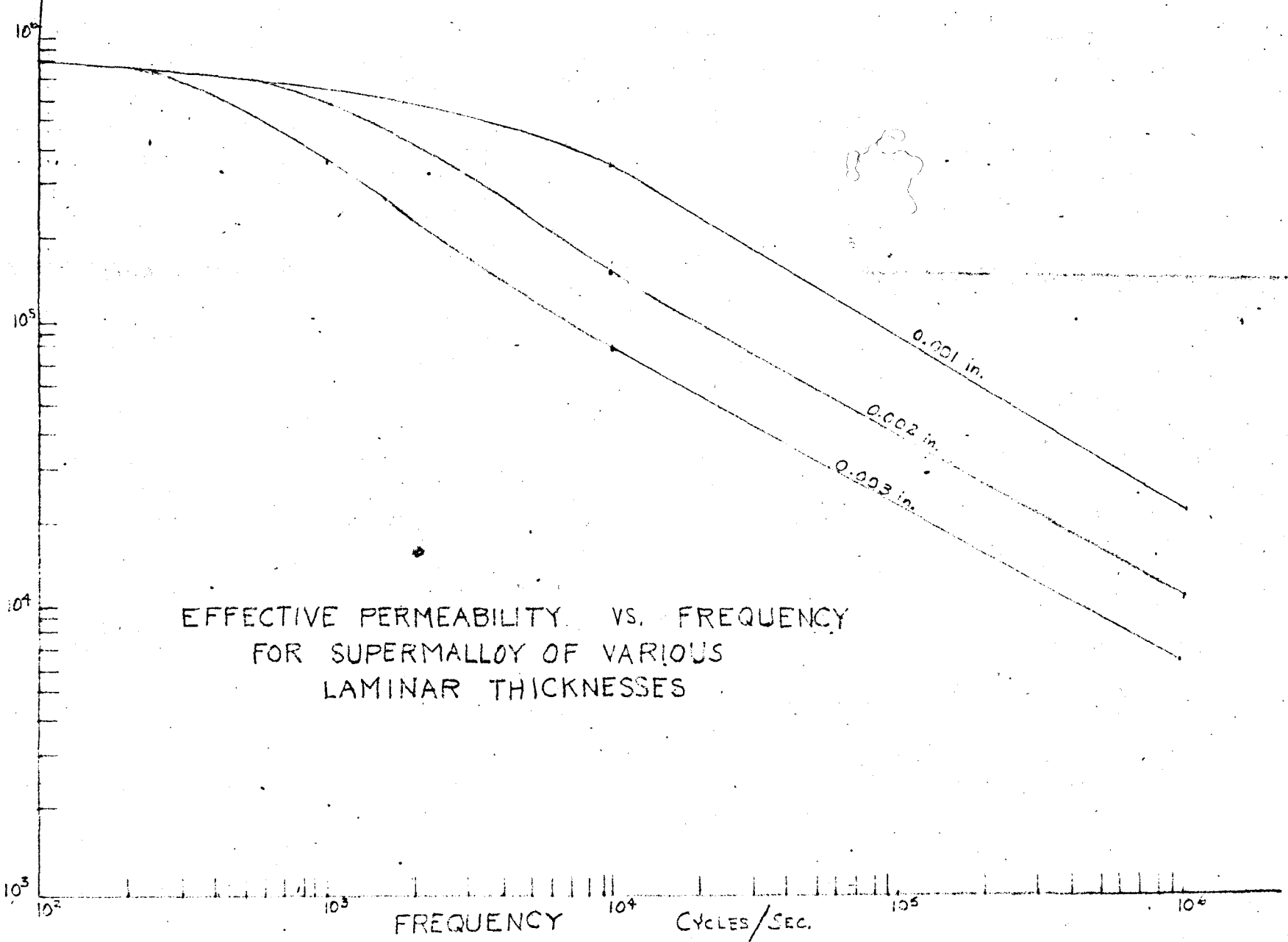
If the apparent permeability is plotted as a function of frequency for the various types of magnetic materials, there is a rapid decline of permeability with frequency up to the region of about 2 megacycles, but thereafter the curve appears to settle down to either a constant value of permeability or a slow decrease of permeability



EFFECT OF EDDY CURRENT SHIELDING
ON APPARENT PERMEABILITY AND
ON EDDY CURRENT RESIST./REACTANCE
RATIO ON SHEET CORE MATERIAL.

FIGURE 12

EFFECTIVE PERMEABILITY



EFFECTIVE PERMEABILITY VS. FREQUENCY
FOR SUPERMALLOY OF VARIOUS
LAMINAR THICKNESSES

FIGURE 13.

with frequency, as shown by Dannatt (8).

The late theories of ferromagnetism suggest that ferromagnetic materials consist of magnetic regions or domains which are in a saturated state without any external magnetic field applied. Each domain has its axis of magnetization different from its surrounding domains. These domains are estimated to have linear dimensions of the order of 10^{-3} to 10^{-4} cm. When an external field is applied to the ferromagnetic material, these domains line up parallel to each other in the direction of easy magnetization (i.e., the 100 axis of the crystal) nearest that of the applied field. This condition corresponds to the steep portion of the magnetization curve (see Fig. 1). As stronger fields are applied, the domain flux comes into more exact parallelism with the applied field and consequently into less permeable directions (i.e. into the 111 axis of the crystal). This condition would correspond to the knee of magnetization curve. Finally as all the domains become lined up with the applied magnetic field the saturated or upper flat region of the magnetization curve is reached. Many high permeability magnetic materials are grain orientated, that is, the crystals are forced to form in the material in the desired directions. This is accomplished by a cold rolling process and controlling the heating and cooling cycle during the annealing process.

Dannatt (8) explains the decrease in permeability with frequency as due to rapidly alternating fields which cause the magnetic domains to be subjected to a type of molecular viscosity during the lining-up process. It might be expected under these conditions that the value of permeability under high frequency magnetization would reach a limiting value with increasing frequency, equal to the initial permeability of the material when measured at low frequency. This initial permeability is the permeability in the absence of any lining-up of the magnetic domains.

Dannat (8) finds however, in actual experiments, the final or ultimate value of the permeability is considerably lower than the initial permeability. This is accounted for by the decreased permeability of the outside layer of domains of a material. When the depth of penetration of flux into the material reaches the magnitude of the linear dimensions of the domain, approximately 0.001 cm., the expected value of permeability becomes small compared to the initial permeability. If the frequency is increased still further, the permeability should remain nearly constant since demagnetization effects relates to the domains in the outermost layer. Since the permeability is practically independent of frequency above about two megacycles, it follows that the rotational effects in the domains have become unimportant when the depth of penetration of the

magnetic flux becomes critical. This depth of penetration is expressed as:

$$d = \frac{\sqrt{2}}{2\rho} = \frac{\sqrt{2}}{4\pi} \sqrt{\frac{\rho}{\mu \cdot f}} \quad \text{absolute units} \quad (20)$$

The effect of eddy currents is to prevent the field from penetrating immediately to the interior of the material, and when the applied field is varying continually, the field strength in the interior of the material may never be more than a small fraction of the field strength at the surface. The magnetic induction, therefore, decreases from the surface toward the interior. As expressed in Equation (10), the effective permeability is:

$$\mu = \frac{[\mu, \rho]^{\frac{1}{2}}}{2\pi t f^{\frac{1}{2}}} \quad (10)$$

In a magnetic amplifier working at radio frequencies, the constants of the materials used may be such that 95 per cent of the true permeability is lost. At these high frequencies the flux carrying ability of a material of given resistivity and thickness is directly proportional to the square root of its permeability.

CHAPTER IV

THE APPLICATION OF RADIO FREQUENCIES TO MAGNETIC AMPLIFIERS

1. Method of Operation.

While magnetic core materials have been used at radio frequencies, such as in intermediate frequency transformers and radar pulse transformers, the use of magnetic amplifiers at radio frequencies has not been investigated to any great degree. The most foreboding problem, as previously mentioned, is that of maintaining reasonable permeability in order that control may be retained. The present day crystal diode type of rectifiers, while not possessing ideal characteristics, has proved itself suitable in most cases for use even at micro-wave frequencies. Therefore, the main body of this chapter will be devoted to the investigation of suitable magnetic materials and circuits in which they are to be used.

Since the change of anode current through the load resistance varies as the permeability of the core material changes, it is obvious that better control or gain per stage can be obtained if a large change in permeability can be produced. Thus, the sharper the knee of the saturation curve, the higher the degree of control that can be realized. However, the knee of the magnetization curve

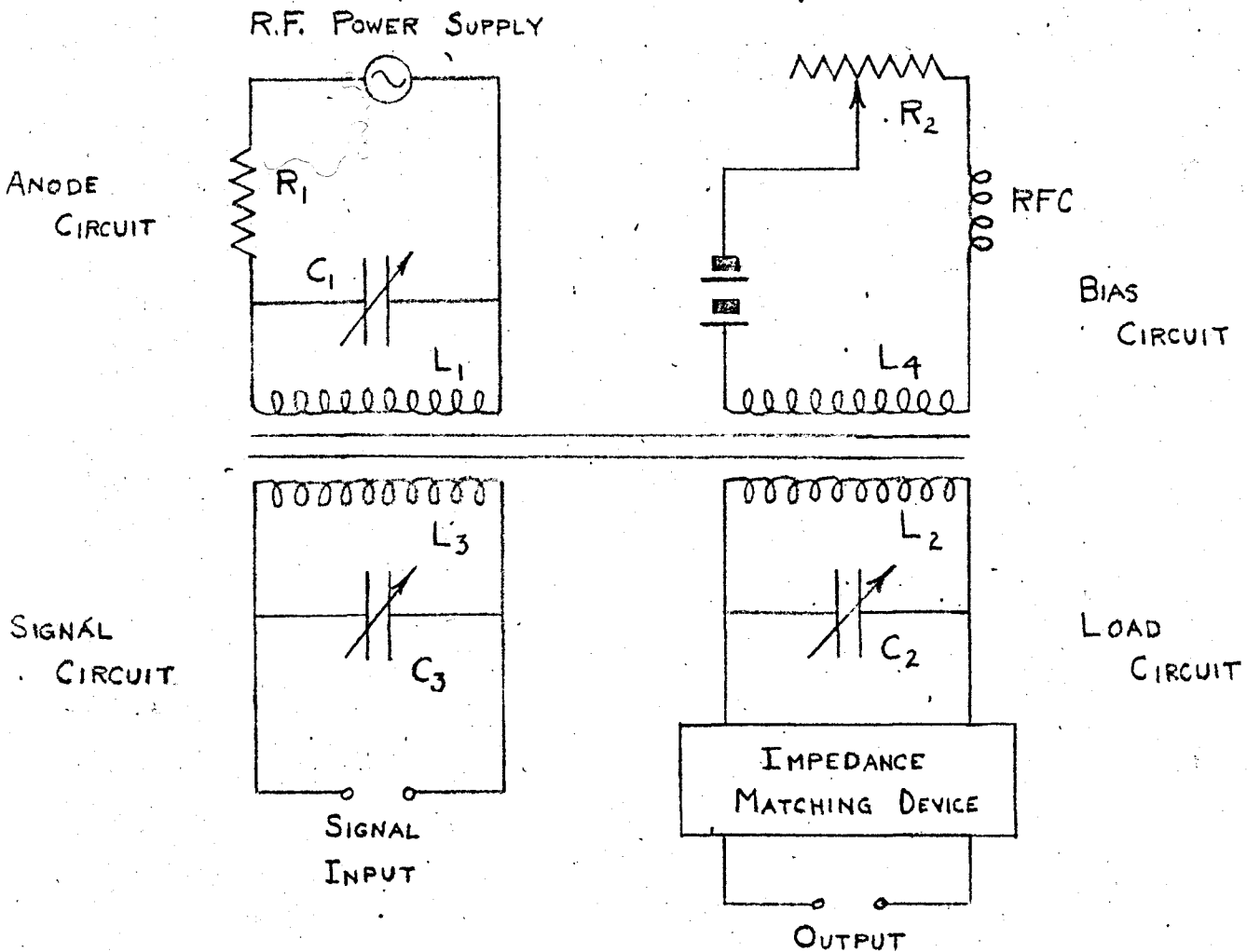
at radio frequencies is rather broad compared to the sharp knee at power frequencies; the conventional magnetic amplifier circuits would give comparatively poor sensitivity to control current. Hence, the circuit shown in Fig. 14 was devised to obtain better sensitivity.

The four windings L_1 , L_2 , L_3 , and L_4 were wound on a closed toroidal Supermalloy* core. L_1 was tuned to resonance of the R.F. power supply frequency with C_1 . R_2 is adjusted so the current flowing in the Bias Circuit changes the permeability of the core, thus changing the value of L_1 and detuning the L_1C_1 combination. This establishes the quiescent operation point in the center of the linear region on the side of L_1C_1 resonance curve. See Fig. 15. A radio frequency choke is added to prevent R.F. current from flowing in the Bias Circuit.

If an alternating signal voltage is now applied to the Signal Circuit which is tuned to the signal frequency by means of L_3C_3 , the value of L_1 will vary about the value established by the D.C. bias current. L_1C_1 will thus approach resonance when the signal current is positive, for example, and move further from resonance when the signal current is negative, as indicated in Fig. 15.

The effect then is to move f_0 or the resonance curve of Fig. 15 back and forth along the horizontal frequency axis about some fixed frequency. This is essentially a

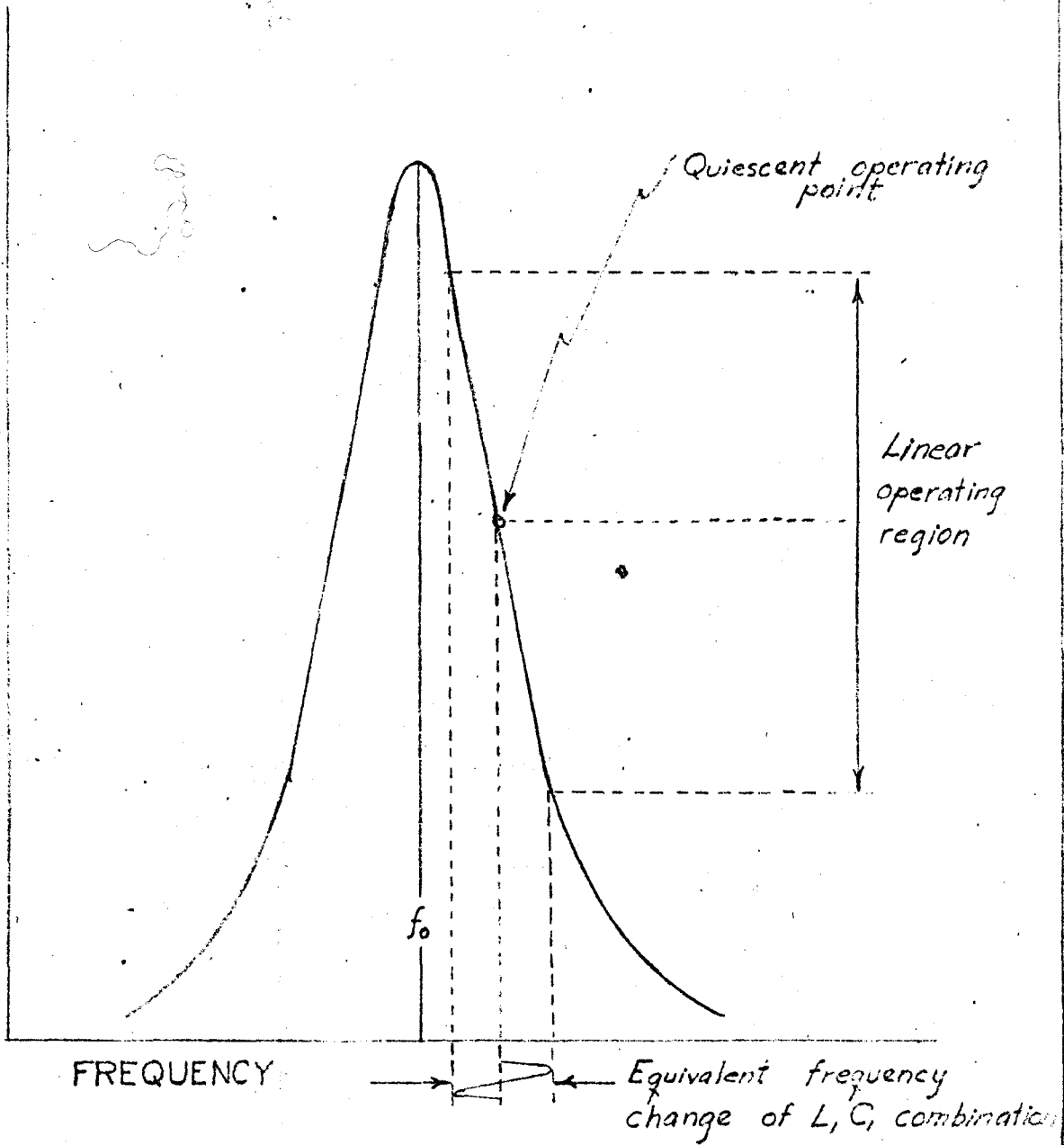
* Bell Telephone Laboratories, New York, New York.



MAGNETIC AMPLIFIER FOR RADIO FREQUENCY SIGNALS

FIGURE 14.

VOLTAGE ACROSS L_1



RESONANCE CURVE AND OPERATING
CONDITION FOR $L, C,$
COMBINATION

FIGURE 15.

non-linear action which modulates the R.F. power supply signal and the voltage appearing across L_1C_1 will appear as an ordinary modulated radio frequency carrier signal.

The modulated output may be removed from the circuit as a voltage across R_1 or by the Load Circuit of Fig. 14. L_2 and C_2 may be tuned to the R.F. output frequency, or without C_2 the voltage may be taken off the L_2 windings by transformer action. As previously mentioned, the load must be matched to the output circuit.

Providing the signal to be amplified is an unmodulated radio frequency signal, the output across the load resistor will be:

$$e = E_o [1 + m \sin \omega_c t] \sin \omega_p t$$

$$= E_o \left[\sin \omega_p t + \frac{m}{2} \cos(\omega_p - \omega_c)t - \frac{m}{2} \cos(\omega_p + \omega_c)t \right] \quad (21)$$

where ω_c is the signal frequency (carrier), ω_p is the power supply frequency, and m the degree of modulation.

If however, the input signal is an amplitude modulated radio frequency signal, then the output of the amplifier is a doubly modulated wave:

$$e = E_o \left[\sin \omega_c t + \frac{m}{2} \cos(\omega_c - \omega_a)t - \frac{m}{2} \cos(\omega_c + \omega_a)t \right] \sin \omega_p t \quad (22)$$

where ω_a is the first signal or audio frequency, ω_c is the signal carrier frequency, and ω_p is the power supply frequency. Multiplying Equation (22) out, gives:

$$e = \frac{E_0}{2} \left[\cos(\omega_p - \omega_c)t - \frac{m}{2} \sin(\omega_p - \omega_c - \omega_a)t + \frac{m}{2} \sin(\omega_p - \omega_c + \omega_a)t - \cos(\omega_p + \omega_c)t + \frac{m}{2} \sin(\omega_p + \omega_c - \omega_a)t - \frac{m}{2} \sin(\omega_p + \omega_c + \omega_a)t \right] \quad (23)$$

This voltage gives summation and difference frequencies with appropriate side bands, similar to the output of the first detector of a superhetrodyne receiver.

If a second amplifier stage of a similar type were cascaded, utilizing the same power supply, the summation and difference frequencies will again result with an output of a summation signal of twice the power supply frequency and a difference signal of the original signal carrier (the power supply frequency being suppressed).

Thus, it may be seen that several stages of this type may be cascaded by tuning each succeeding stage to the proper frequency. It must be born in mind, however,, that if a succeeding stage were tuned to the summation frequency, the difference frequency component would be lost. This system would inherently give only half the

power gain of which each stage would be capable of producing.

In this type of amplifier circuit, the sensitivity and gain will be a function of the Q obtainable. Neglecting the resistance of the winding L_1 (its loss being small compared to the core losses); the Q of a tuned circuit will be a function of hysteresis and eddy current losses. As has already been pointed out, the hysteresis loss, which is a linear function of frequency, is small compared to eddy current loss which varies as the second power of the frequency.

The effort to improve the amplifier performance can then be directed to reducing core losses by thinner laminated and higher resistivity materials. The Supermalloy used in this case was the best obtainable material, in that its maximum permeability was still a reasonable value at one megacycle. Some powdered cores, such as Ferroxcube* which are composed of fine grains of high permeability material, indicate that they may possibly be the answer to the core loss problem. The difficulty with the powdered core is the excessive flux leakage from the infinite number of small air gaps between the ferrite particles. This flux leakage decreases the effect of control,

* North American Philips Company, New York, New York.

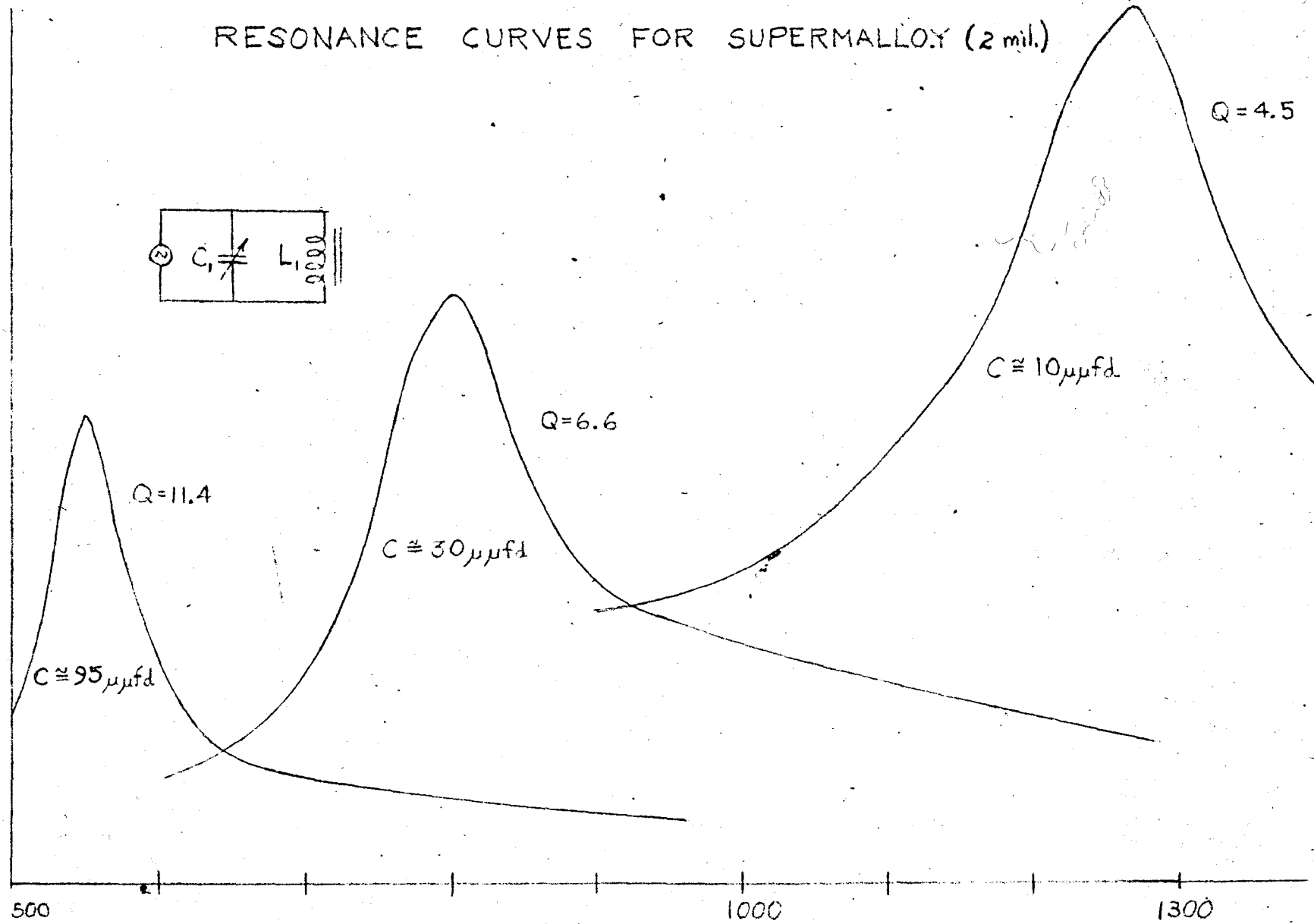
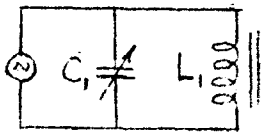
since a large portion of the flux set up by the control windings escapes the core, thus preventing it from controlling the magnetic flux of the power supply. The Ferroxcube core will give Q's in the order two hundred which indicates that a compromise might be reached between flux leakage and eddy current losses.

The values of maximum permeability, as a function of frequency, are plotted in Fig. 13 for Supermalloy. Measured values of Q's for various frequencies are plotted in Fig. 16 for Supermalloy. As previously discussed in the work done by Dannatt (8), the permeability remains nearly constant from frequencies of two megacycles upward toward six hundred megacycles. Providing the value of permeability in this "flat range" can be made a reasonable value, then an amplifier might be used well into the radio frequency spectrum, providing also that the permeability could be changed with control flux.

The gain of the circuit in Fig. 14 can be computed as a power ratio or in db. The input power will be that dissipated in the equivalent resistance of the tuned circuit. The output power will be that contained in the side bands of the modulated wave. The final expression for power gain will be: (See Appendix A)

RESONANCE CURVES FOR SUPERMALLOY (2 mil.)

VOLTAGE ACROSS L_1 VOLTS



FREQUENCY KILOCYCLES PER SECOND
FIGURE 16

$$\text{Power Gain} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{m^2}{2} \frac{E_{\text{carrier}}^2}{E_{\text{signal}}^2} \frac{L_3 R_1' C_1 (1 + 4Q_1^2 \delta^2)}{R_3 C_3 [R_1 R_1' C_1 (1 + 4Q_1^2 \delta^2) + L_1]} \quad (24)$$

where

$$\delta = \frac{\omega}{\omega_0} - 1 = \frac{f - f_0}{f_0}$$

The power gain may thus be maximized by using one hundred per cent modulation and keeping the L to C ratios of the tuned circuits as high as possible. This may be accomplished practically by using the distributed capacity of the windings and as a small a variable capacity as possible. The use of Litz wire for the windings will minimize R_1 , and R_3 , the winding resistances. Omitting R_1 and removing the amplified signal from the Load Circuit will increase the gain somewhat depending on the amplitude of the load current. Since the maximum values of L_1 and L_3 will be functions of frequency, the power gain is also a function of frequency as expressed by Equations (1) and (9).

For amplification without distortion, the equivalent frequency shift caused by the signal flux must not cause the operating point to go beyond the linear region of the

resonance curve. See Fig. 12. Thus for distortionless amplification, the input signal is limited by the Q of the Anode Circuit.

2. Experimental Results.

The circuit shown in Fig. 14 was erected using a core wound of Supermalloy tape of two mill laminar thickness. The radio frequency power supply consisted of a 6L6 tube, electron coupled oscillator driving an 807 tube. The plate of the 807 vacuum tube was coupled to the anode winding through a capacitor. The anode winding L_1 , which consisted of forty turns of Litz wire, and the variable capacitor C_1 formed the output tuned circuit to the 807 vacuum tube. The plate power supply voltage for the 807 tube was variable from zero to 650 volts.

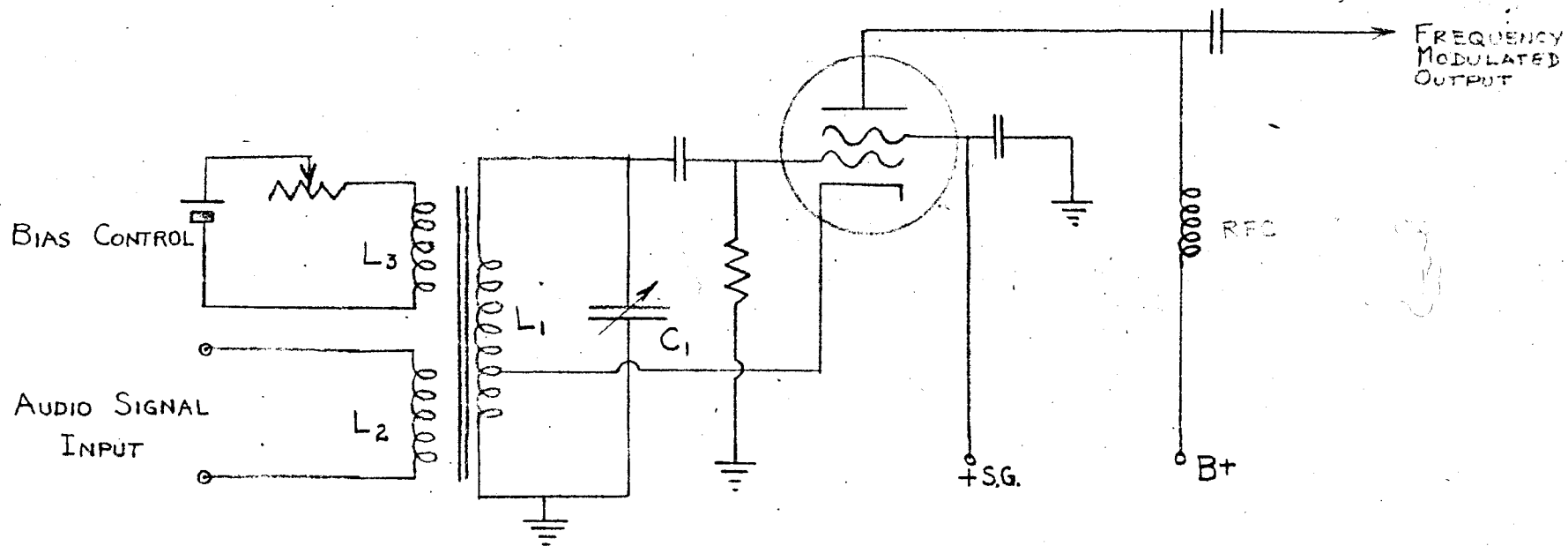
The bias supply consisted of a nine volt dry cell battery, which was connected through a two and one-half milli-henry radio frequency choke to the windings of L_4 . The bias windings consisted of 425 turns of number thirty-two B. and S. gauge wire.

The signal input windings consisted of one hundred turns of number thirty-two wire and the load windings consisted of fifty turns of the same gauge wire. All voltages were measured with a high impedance vacuum tube voltmeter and wave forms were observed with a cathode-ray oscilloscope for distortion.

At power supply frequency of 500 kilocycles and a signal frequency of 50 kilocycles the power gain was found to be 1120 times. See Appendix B. However, since the permeability and Q of the core and associated circuits are a function of frequency, the gain will vary with the signal and power supply frequencies. These variations were measured and are plotted in Figs. 18 and 19. Fig. 18 shows the power gain as a function of power supply frequency. Fig. 19 shows the power gain as a function of signal frequency. In all measurements made, the wave forms were observed to keep the amplification as nearly linear as possible. Since the oscilloscope will not show small amounts of distortion, these results indicate the amplifier's general characteristics.

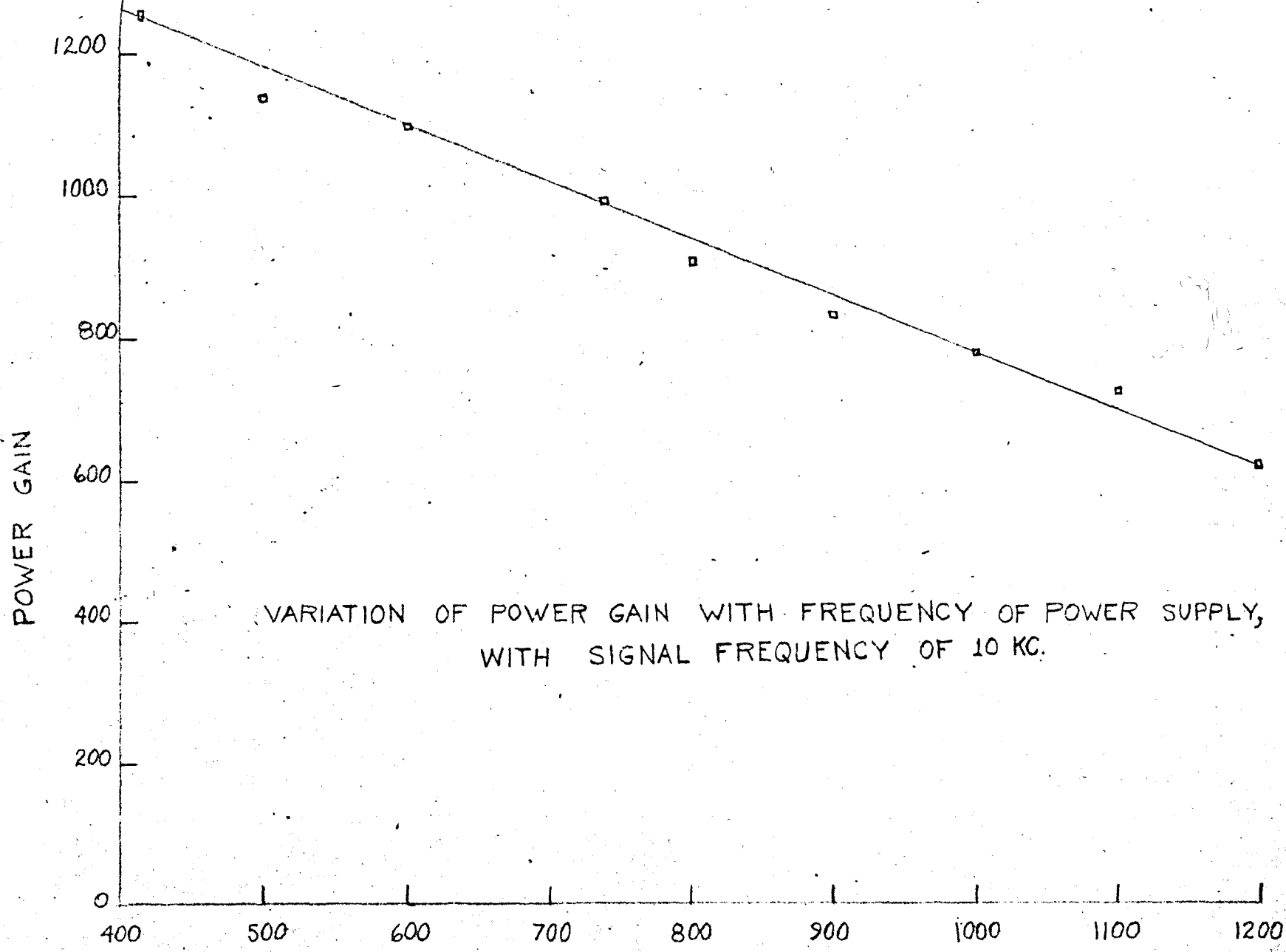
3. Other Uses of the Magnetic Amplifier at Radio Frequencies.

Another use for the magnetic amplifier is that of amplitude modulation for a radio transmitter. E. F. W. Alexanderson's original magnetic amplifier was designed specifically for this purpose. However, this modulator was designed when very low radio frequency carriers were used. In the future as better magnetic materials are developed, the return to the magnetic amplifier as a modulator will become feasible and can compete with the types of amplitude modulation employing vacuum tubes.



UTILIZATION OF A MAGNETIC AMPLIFIER TO
 FREQUENCY MODULATE AN ELECTRON
 COUPLED OSCILLATOR

FIGURE 17.



VARIATION OF POWER GAIN WITH FREQUENCY OF POWER SUPPLY,
WITH SIGNAL FREQUENCY OF 10 KC.

POWER SUPPLY FREQUENCY KC
FIGURE 18

VARIATION OF POWER GAIN WITH SIGNAL FREQUENCY,
WITH POWER SUPPLY FREQUENCY OF
1100 KC.

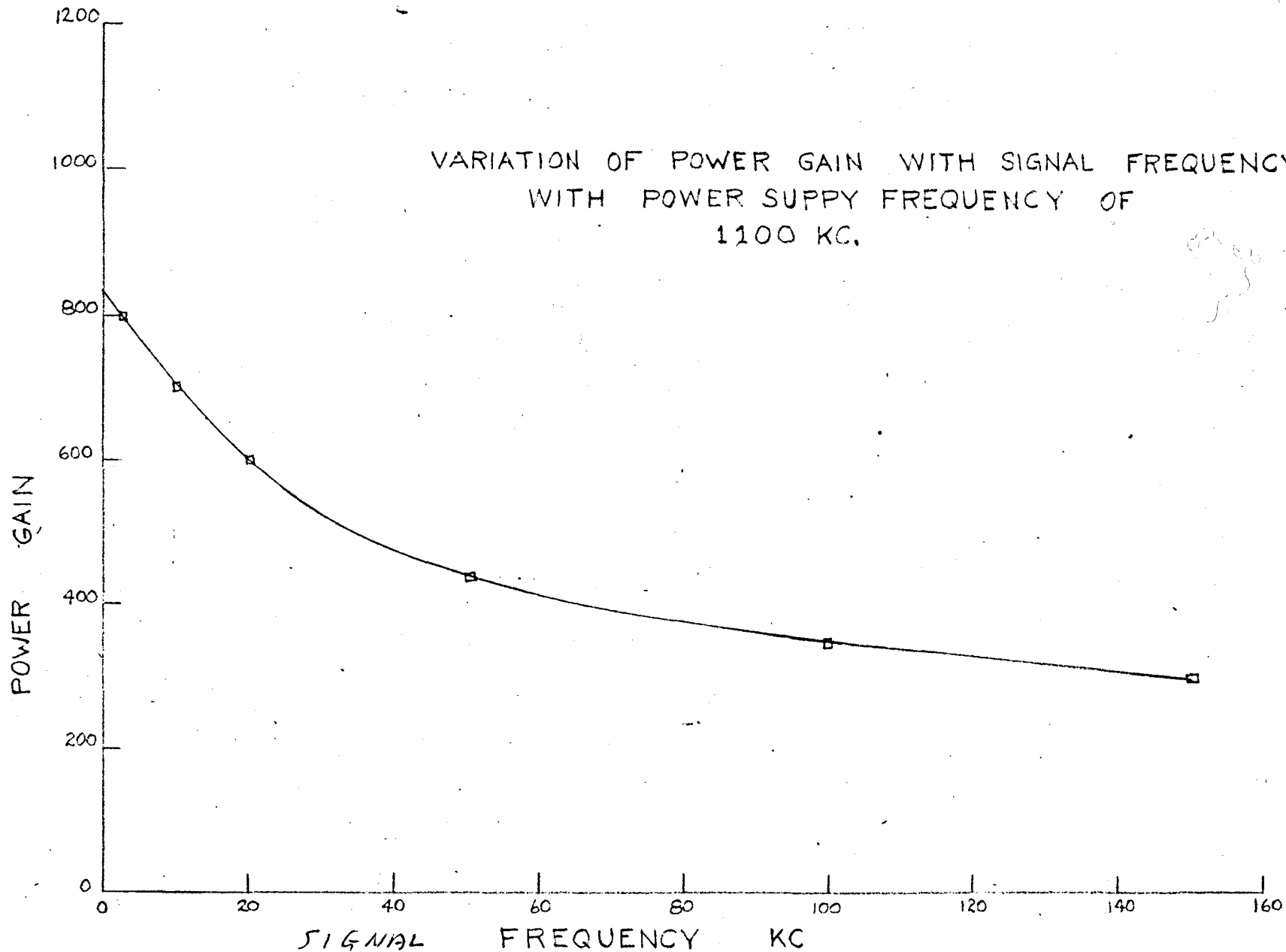


FIGURE 19

Various types of modulation circuits can be used employing the magnetic amplifier, as modulation in the plate or grid circuits. With the present type of magnetic materials now available however, the core losses make plate modulation inefficient.

Frequency Modulation may also be accomplished by using the magnetic amplifier in the same manner as a reactance tube. As in Fig. 17, the grid tuned circuit inductance is wound on a high permeability core with auxiliary windings for bias current and the modulating audio signal. L_1 and C_1 are tuned to the center frequency which is multiplied in succeeding stages of vacuum tube amplification. The value of L_1 is a function of the core's permeability as given in Equation (1). The bias current is adjusted so that the core is operating in a linear region of its permeability curve, (see Fig. 1). Any flux now added to the core will change its permeability and also the inductance of L_1 . This change of permeability, caused by the additional flux of the audio signal, will cause the tuned circuit, L_1C_1 , to shift the frequency of the oscillator. The oscillator frequency then becomes a linear function of the amplitude of the audio signal. The output frequency spectrum of the oscillator becomes the same as the output of an oscillator whose frequency is determined by a reactance tube in the grid circuit.

As previously mentioned, audio amplifiers* have been built and successfully demonstrated. These amplitude modulate supersonic frequency waves with audio frequency signals. The advantages of such an amplifier are dependability, ruggedness, and the ability to handle more power of audio signal per pound of amplifying equipment. The main disadvantage is in obtaining a supersonic or radio frequency power source which can be converted into audio power. Except for the power source, and by use of dry or crystal type rectifiers, the entire amplifier can be made without the use of vacuum tubes. While this characteristic is not a great advantage in many instances, for Naval uses where dependability is a necessity, it can prove superior to the vacuum tube amplifier. Another advantage of this type of amplifier, both at radio and audio frequencies, is the low noise level. The only inherent noise of a magnetic amplifier is the "Barkhausen effect" of noise caused by the sudden shifts of the magnetic domains within the magnetic materials. This effect gives a random sharp peaked noise similar to vacuum tube "shot noise" but at a lower power level than the "shot effect" found in vacuum tubes. In high permeability magnetic materials in particular, is this noise level low.

*Ward Leonard Electric Company, Mount Vernon, New York.

The future of the magnetic amplifier at radio frequencies will depend on whether or not physical means can be found to reduce the laminar thickness of the magnetic materials. New methods in producing powdered and sintered cores, promise materials with extremely low eddy current losses, which will give materials whose permeabilities will remain high well into the radio frequency spectrum.

In this discussion it has been assumed that the true permeability is independent of frequency and that only the apparent permeability diminishes as the frequency increases. There is some reason to believe; however, that at ultra-high frequencies, the magnetic domain is not capable of changing with the required rapidity. R. Becker (5) has calculated the order of the magnitude of the frequency at which the magnetization should fail to respond. He estimates that the permeability will begin to decrease seriously when the frequency is about two thousand megacycles, corresponding to a wave-length in the air of fifteen centimeters.

While this calculation may indicate the upper limit for the magnetic amplifier, there is still a considerable portion of the radio frequency spectrum in which the magnetic amplifier may be used.

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APPENDIX A

The power input to the signal circuit may be expressed as:

$$\text{Input Power} = P_{in} = \frac{E_{\text{signal}}^2}{R_{\text{equivalent}}} = E_{\text{signal}}^2 \frac{R_3 C_3}{L_3} \quad (25)$$

The output power contained in the side bands of the modulated wave is:

$$\text{Output Power} = P_{out} = \frac{m^2}{2} P_{\text{carrier}} \quad (26)$$

where m is the degree of modulation; E is the rms value of the applied voltage; R , L , and C are the values of resistance, inductance, and capacitance of their respective circuits.

The carrier power, P_{carrier} , may be expressed by:

$$P_{\text{carrier}} = \frac{E_{\text{carrier}}^2}{R_e(Z_1)} \quad (27)$$

where $R_e(Z_1)$ is real part of the impedance of the anode circuit. Since $L_1 C_1$ are not tuned to exact resonance, the impedance Z_1 is:

$$Z_1 \cong R_1 + \frac{L_1}{R_1 C_1} \frac{1}{1 + j 2 Q_1 \delta} \quad (28)$$

where

$$\delta = \frac{\omega}{\omega_0} - 1 = \frac{f - f_0}{f_0}$$

The real part of Z_1 is then:

$$\text{Re}(Z_1) \cong R_1 + \frac{L_1}{R_1 C_1} \frac{1}{1 + 4 Q_1^2 \delta^2}$$

and the carrier power becomes:

$$P_{\text{carrier}} = \frac{E_{\text{carrier}}^2}{R_1 + \frac{L_1}{R_1 C_1} \frac{1}{1 + 4 Q_1^2 \delta^2}} \quad (29)$$

The power gain which is the ratio of the output power to the input power, is:

$$\text{Power Gain} = \frac{m^2}{2} \frac{E_{\text{carrier}}^2}{E_{\text{signal}}^2} \frac{L_3 R_1 C_1 (1 + 4 Q_1^2 \delta^2)}{R_3 C_3 [R_1 R_1 C_1 (1 + 4 Q_1^2 \delta^2) + L_1]} \quad (24)$$

APPENDIX B

The following typical data was observed. The gain may be calculated from this data by Equation (24).

m,	0.80
E_{carrier} ,	200 volts.
E_{signal} ,	5 volts.
f_{carrier} ,	5×10^5 cps
f_{signal} ,	5×10^4 cps
C_1 ,	26 mmfd.
R'_1 ,	0.723 ohms
R_1 ,	2×10^4 ohms
R_3 ,	2.0 ohms
C_3 ,	168 mmfd.
Q_1 ,	12.6
N_1 ,	40
N_2 ,	50
N_3 ,	100
N_4 ,	425
A,	0.98 sq. cm.
l ,	10.4 cm.
δ ,	0.052

From Fig. 13, $\mu = 1.5 \times 10^4$. Then:

$$L = \frac{4\pi N^2 A \mu}{\ell} \times 10^{-9} \quad \text{heneries} \quad (1)$$

$$L_1 = 2.54 \times 10^{-2} \text{ heneries}$$

$$L_3 = 1.65 \times 10^{-1} \text{ heneries}$$

$$\text{Power Gain} = \frac{m^2}{2} \frac{E_{\text{carrier}}^2}{E_{\text{signal}}^2} \frac{L_3 R_1' C_1 [1 + 4Q_1^2 \delta^2]}{R_3 C_3 [R_1 R_1' C_1 (1 + 4Q_1^2 \delta^2) + L_1]} \quad (24)$$

$$= \frac{0.64}{2} \cdot \frac{4 \times 10^4}{25} \frac{(0.165)(0.72)(2.6 \times 10^{-11}) [1 + (4)(150)(0.052)]}{(2)(1.68 \times 10^{-10}) [(2 \times 10^4)(0.72)(2.6 \times 10^{-11}) (1 + (4)(150)(0.052)) + 2.54 \times 10^{-2}]}$$

where,

$$\delta = \frac{\omega}{\omega_0} - 1 = \frac{510}{500} - 1 = 0.02$$

Power Gain = 1120

$$= 10 \log 1120 = (10)(305) = 30.5 \text{ db}$$

Other values of power gain have been computed and are plotted as a function of power supply frequency in Fig. 18 and as a function of signal frequency in Fig. 19.