THE DESIGN OF MAGNETIC AMPLIFIERS; A MAGNETIC AMPLIFIER FOR RADAR ANTENNA CONTROL

BY

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PREFACE

The writer has described a practical method of magnetic amplifier design. Some of the problems associated with any method of design have been considered. A special problem is solved in Chapter II.

The writer acknowledges valuable advice from Reuben Lee, C. K. Hooper, and J. W. Evans, engineers of the Westinghouse Corporation, Baltimore, Maryland, on the design problem of Chapter II.

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SYMBOLS

Chapter I

SYMBOL	DESCRIPTION
I_c	Control Current
ϕ	Flux
MMF	Magnetomotive force
B_f	Flux density value at firing
B_o	Initial flux density
N	Number of turns
ω	$2\pi f$
E_m	Maximum value of a-c voltage
ϕ_m	Maximum value of a-c flux
B_m	Maximum value of a-c flux density
θ_f	Angle before firing
I_A	Current in winding A
I_B	Current in winding B
L	Inductance
R	Resistance
I_{ac}	Alternating Current
X_L	Inductive reactance
μ	Permeability
A	Area of core
l	Mean length of core
$\hat{B}$	Peak value of flux density
i_n	n^{th} harmonic
Z	Impedance

SYMBOL	DESCRIPTION
k_1, k_2	constants
β	Feedback factor
$(NI)'_{dc}$	Total d-c mmF
I_{fdbk}	Feedback current
N_{fdbk}	Number of feedback winding turns
e	Instantaneous value of voltage

Chapter II

SYMBOL	DESCRIPTION
RPM	Revolutions per minute
e_g	Counter electromotive force
k_v, k_T	constants
ω	Radians per second
t_m	Torque in lb-ft.
e	Applied d-c voltage
T_L	External load in lb-ft.
i	Instantaneous armature current
R_a	Armature resistance
J	Inertia of armature and its load in slug-ft.^{2}
H.P.	Horsepower
V_{ac}	a-c voltage across rectifiers
V_a	Armature voltage
I_l	Load current
R	Resistance in parallel with armature

SYMBOL	DESCRIPTION
V_{app}	Applied a-c voltage
E	Voltage across reactor
$I_{dc\ max}$	Maximum value of control current
V_{rect}	Voltage across reactor
I_a	Armature current
B_f	Flux density value at firing
B_o	Initial flux density
θ_f	Angle before firing
R_c	Control winding resistance
I_c	Control current
I'_{dc}	Control current value with no feedback
e_L	Load voltage
I_d	Current in regulating winding
V_d	Voltage to balance V_a at full load
P_1	Speed control potentiometer
P_2	Regulation control potentiometer
R_d	Resistor that limits I_d

INTRODUCTION

The magnetic amplifier is a device which may be inserted between a power source and a load to vary the amount of voltage appearing across the load. It may be thought of as a variable impedance in series with the source and the load. Varying this impedance will result in a variation of the voltage across the load. Magnetic amplifiers now have many applications, and new uses are constantly being found. The problem of how to design a magnetic amplifier for a particular application has become pressing. An attempt is made in Chapter I to explain some of the problems involved in magnetic amplifier design and to present a method that can be used to solve the design problem in a simple way.

Chapter II deals with a specific problem--the design of a magnetic amplifier to control a radar antenna. The amplifier was designed according to the procedure outlined in Chapter I, and it operated successfully. It should be noted that the conditions imposed on the magnetic amplifier of Chapter II were rather severe. The load varied over a wide range, and yet this load variation was not to be reflected in a motor speed change. Such a problem would have required complicated circuitry and numerous tubes in the conventional antenna controller. The magnetic amplifier needed no tubes and very few circuit components.

Appendix A takes up the question of tracking. Here the magnetic amplifier seems to fail as a substitute for conventional circuits. The reason for this stems primarily from the fact that no one as yet has discovered a configuration of cores and coils that will track without excessive losses and excessive weight.

Chapter I

MAGNETIC AMPLIFIER DESIGN

1. Simple Magnetic Amplifier

Perhaps the simplest type of magnetic amplifier is illustrated in Figure 1. It is necessary, in practical applications, to use two cores, two load (a-c) windings, and two control windings to make possible cancellation of the effects of the load windings on the control winding by transformer action. A single three-legged or four-legged core may be used if available. Such a core does not differ, essentially, from the arrangement of Figure 1. The load windings may be either in series or parallel, one arrangement being somewhat superior to the other for particular applications. The control windings are always arranged in the manner shown in Figure 1. Such an arrangement causes a concellation of the voltages induced in the control windings by currents in the load windings.

The load, the load windings and the a-c supply are in series in Figure 1. Any variation in reactance of the load reactors will cause a variation in the voltage appearing across the load. Increasing the control current causes a decrease in the impedance of the load windings and a corresponding increase in the voltage across the load. Decreasing the control current has the opposite effect. The variation of impedance may be shown graphically by the method depicted in Figure 2.

Figure 2 shows a sinusoidal voltage across the reactors. This voltage causes a sinusoidal flux which lags the voltage by ninety degrees. It may also be said that this flux is caused by the

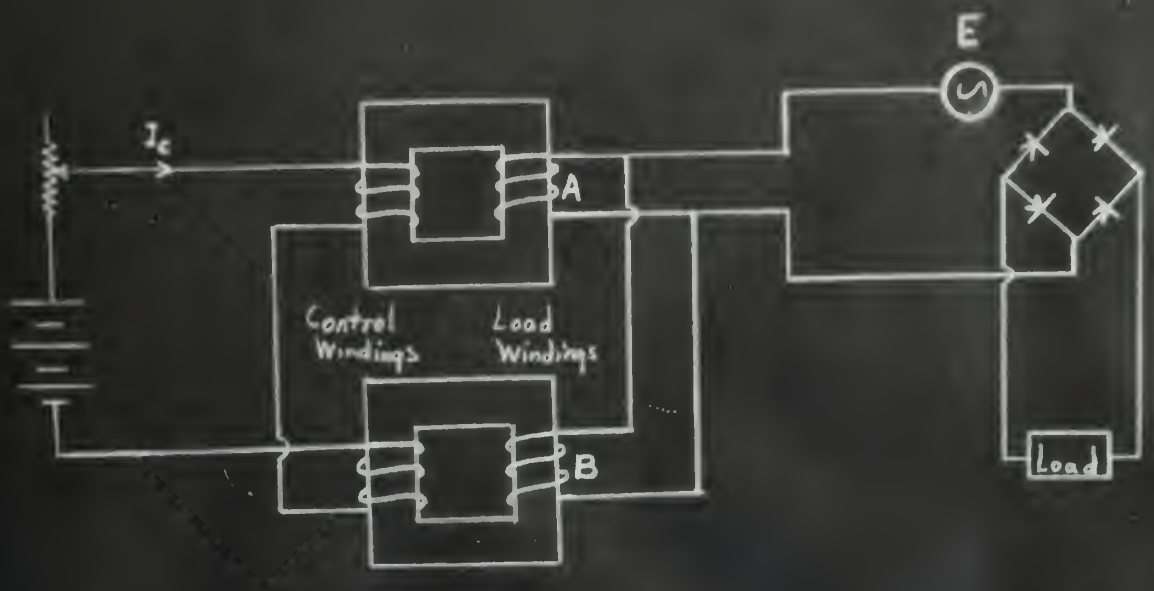


Figure 1 - Simple Magnetic Amplifier

Figure 2 - Projection of Flux onto Saturation Curve to Obtain MMF

magnetomotive force resulting from the total currents flowing in the load windings.

The magnetomotive force may be determined by projecting the flux onto the ϕ (flux) vs MMF curve and then projecting the mmf onto the MMF vs θ (phase) curve. This determination of the magnetomotive force produces the load current as a by-product since mmf equals NI. It is apparent that increasing the control current will cause the flux in each core to swing above the knee of the ϕ vs MMF curve earlier, thus causing a greater mmf and load current. As a matter of fact, the saturation curve is always much more linear and much steeper than shown in Figure 2, and the knee is much more abrupt, with new high-permeability cores. The curve for a high-permeability core causes the load current to be essentially a series of pulses if the flux swings above the knee. With a given voltage applied, the control current magnitude determines whether "firing" will take place.

Figure 3 shows typical firing angles for several values of control current. If the alternating flux never exceeds the value at the knee because the control current is low ("bias" is too great), firing cannot occur. This is equivalent to saying that the inductive reactance is very high. Obviously, increasing the magnitude of the applied voltage will also cause firing. It is possible to determine analytically the angle of conduction, relative to the supply voltage, of the magnetic amplifier in many cases by using certain circuit parameters and the hysteresis loop of the core. In essence, firing of the magnetic amplifier occurs when the instantaneous flux density in the core reaches a critical value which may be called the firing flux density.*

* Dornhoefer [2]

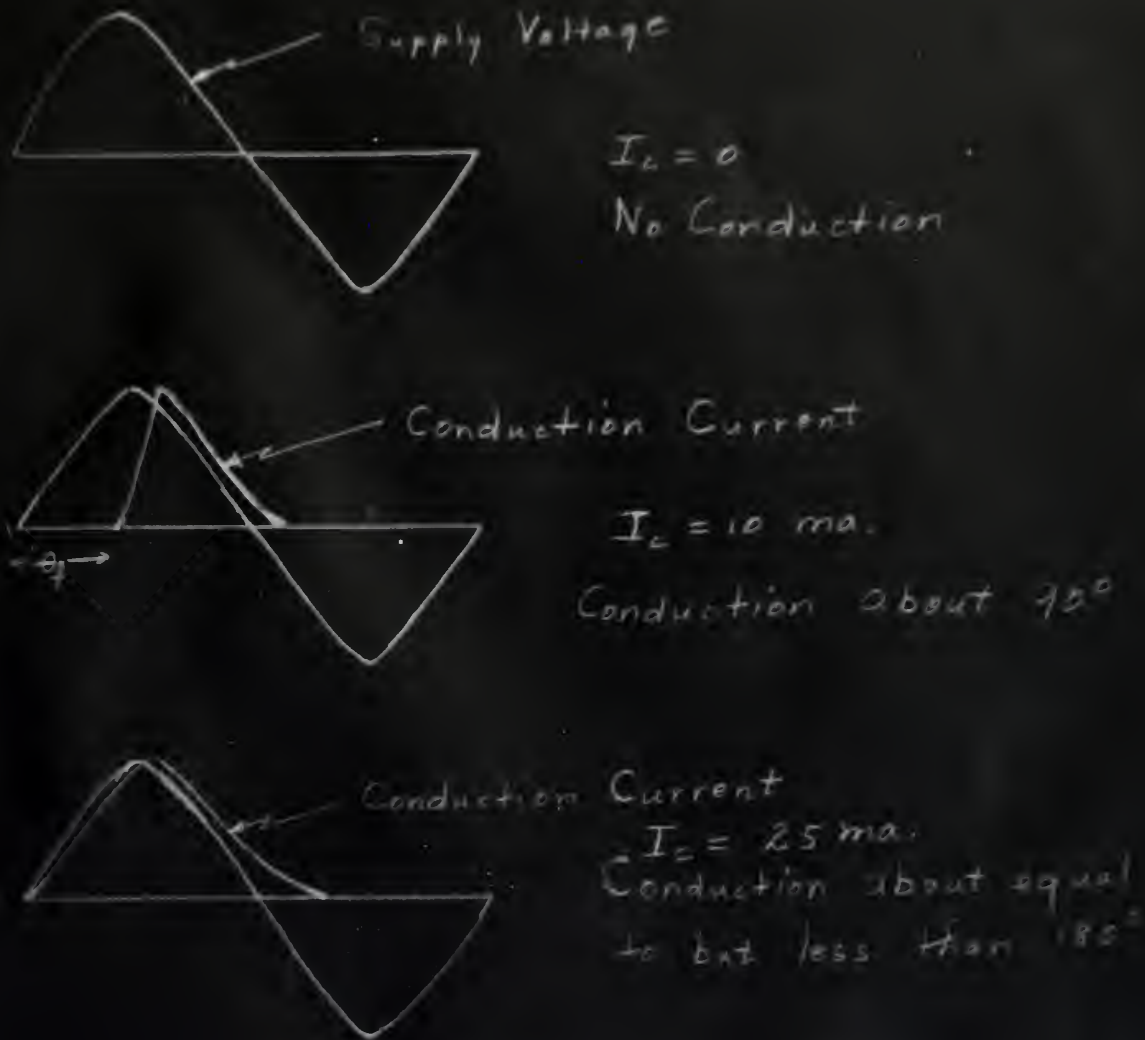


Figure 3 – Conduction of Saturable Reactors

$$\text{Firing angle per reactor} = \cos^{-1}\left[1 - \frac{(B_f - B_o)\,\omega N A}{E_m \times 10^8}\right] \quad (1)$$

$$E = -N \frac{d\phi}{dt} \times 10^{-8} \quad (2)$$

$$\phi = \phi_m \cos \omega t \quad (3)$$

Therefore, $E = N \omega \phi_m \sin \omega t \times 10^{-8}$

$$E_m = N \omega \phi_m \times 10^{-8}$$

$$\phi_m = \frac{E_m \times 10^8}{N \omega}$$

and $$B_m = \frac{E_m \times 10^8}{\omega A N} \text{ gausses} \quad (4)$$

Substituting (4) in (1),

$$\theta_f = \cos^{-1}\left(1 - \frac{B_f - B_o}{B_m}\right) \quad (5)$$

where B_f is the critical or firing value of flux density, and B_o is the flux density in the core due to control current alone. Of course, $0 \leq \theta \leq \pi$ and $-B_f \leq B_o \leq +B_f$. If B_o is less than $-B_f$ or greater than $+B_f$, it is still possible for firing to occur but H_o will be unreasonably large, and the magnetic amplifier will lose much of its responsiveness to control current variations. If B_m exceeds B_f, firing of the amplifier cannot be prevented regardless of the value of B_o. In applications like motor control where it is desired to be able to stop the motor, inability to prevent firing would make it impossible to bring the motor to a stop.

The magnetomotive forces of the two windings were shown graphically in Figure 2. Mathematically, they may be expressed in the form of a series. The same series, divided term-by-term by the number of turns in the windings, represents the currents in the load windings.

$$I_A = I_c + I_1 \cos\theta + I_2 \cos 2\theta + I_3 \cos 3\theta + I_4 \cos 4\theta + \cdots \tag{6}$$

$$I_B = -I_c - I_1 \cos(\theta - 180^\circ) - I_2 \cos 2(\theta - 180^\circ) - I_3 \cos 3(\theta - 180^\circ) - \cdots$$

$$= -I_c + I_1 \cos\theta - I_2 \cos 2\theta + I_3 \cos 3\theta - I_4 \cos 4\theta + \cdots \tag{7}$$

Ic is obtained by dividing the control winding magnetomotive force by the number of turns in the control winding.

The total load winding current is the sum of I_A and I_B, with the windings in parallel.

$$I_{total} = 2(I_1 \cos\theta + I_3 \cos 3\theta + I_5 \cos 5\theta + I_7 \cos 7\theta + \cdots) \tag{8}$$

$$= 2 \text{ (odd harmonic terms)}$$

With load windings in parallel, then, the even harmonic current terms cancel in the load circuit. The odd harmonics of the load winding circuit have no effect on the control windings, even though the harmonics are in phase, because the control windings are reversed in phase. Although there are no even harmonics in the load (or supply) circuit, there very definitely are even harmonics in each of the load windings, as shown in equations (6) and (7). These even harmonics may readily be shared by the control windings. An inductance is sometimes placed in series with the control windings to make their share of these even harmonics small. It is desirable to eliminate these extraneous currents from the control windings since

they constitute an unwanted input to the amplifier. These currents also adversely affect the amplifier's sensitivity.

If it is desired to design a magnetic amplifier with as rapid a response to control current change as possible, the load windings will be in series. Then even harmonic currents can flow only in the control windings. If the even harmonics are permitted to flow in the control windings, the magnetization of the reactors is said to be natural; if the even harmonics are prevented from flowing in the control windings by a large series impedance or some other method, the magnetization of the reactors is said to be constrained since the even harmonics cannot flow at all.* Generally, series load windings are used when fast response is desired and parallel load windings when high gain and high power utilization are the primary considerations. Since the time constant of an inductive circuit is $\frac{L}{R}$, the response time may be decreased by adding resistance in the control windings. This increase of resistance means more power loss in the input circuit, however, and a consequent loss in power gain.

The foregoing discussion of the type of magnetization that results from a particular arrangement of windings is important from the design standpoint. Most methods of design assume a sinusoidal flux in each reactor. This cannot be the case is the magnetization is constrained--if some harmonics cannot flow. Therefore, design is considerably simpler, if it is to be quantitatively accurate, if

* Rex [8]

magnetization is natural. This means all harmonics must flow. As has been indicated, with parallel load windings all harmonics can flow. If they are eliminated in the control windings, they will still exist in the individual load windings so that the flux in each will be sinusoidal. The design problem is consequently simplified by using the type of circuit illustrated in Figure 1.

After it has been decided that the type of circuit shown in Figure 1 is to be used, the next step is to pick out the two reactors. Many high permeability core materials are now available. Among these are hipernik V, orthonik, hipersil, and mu-metal. Assume hipersil is chosen. The size of the core is then determined by the volt-ampere rating necessary as fixed by the maximum load. Window size is important also, for it determines the number of load and control turns that may be used. These turns will usually be wound on the same leg of the core. Figure 4 shows a type "C" core. A banding strap tightly binds the two halves, making the air gap small.

If the load is d-c, the rectifiers may be chosen next. It is important to choose rectifiers with high back resistance, for reverse current will decrease the sensitivity of the amplifier. Selenium rectifiers have been improved to the point where the reverse current is almost negligible. Another factor that must be considered in selecting the rectifiers is the range of voltage that will appear across them and the range of current drown through them. For example, a stack of twenty selenium plates each five inches in diameter will tolerate about three hundred volts a-c across them and will

Figure 4 - Typ[illegible] core

handle a current drain of five or six amperes without getting too hot. The forward d-c voltage drop, however, decreases with current density, particularly with very low values of current density. This is illustrated in Figure 5. The non-linearity for very low current densities means that the voltage available for the load is not a straight-line function of the a-c voltage applied to the rectifier. This apparent difficulty may be obviated by experimentally determining what a-c voltage must be applied to the rectifier to produce the minimum load voltage and current and the a-c voltage necessary to produce the maximum load voltage and current. The a-c voltage necessary over the range will not be linear, since the rectifier characteristic is not linear, but this non-linearity will not prove troublesome in most applications. It is the problem of the designer now to design the magnetic amplifier to produce the required range of a-c voltage and current for the rectifier.

Perhaps the easiest way to design the reactors is to use a graphical method. A family of curves of H_{ac} vs B_{ac} with control current as parameter must be obtained for the hipersil core material. To get these curves it is necessary to wind an a-c and a d-c winding onto each core. Five or six hundred turns of wire for each winding shoud suffice. The circuit of Figure 6 is now set up.

To find the a-c flux density B:

$$I_{ac} = \frac{V_{ac}}{X_L} \qquad (R \text{ is negligible}) \tag{9}$$

$$X_L = 2\pi f L \tag{10}$$

$$L = N\frac{d\phi}{di} \times 10^{-8}$$

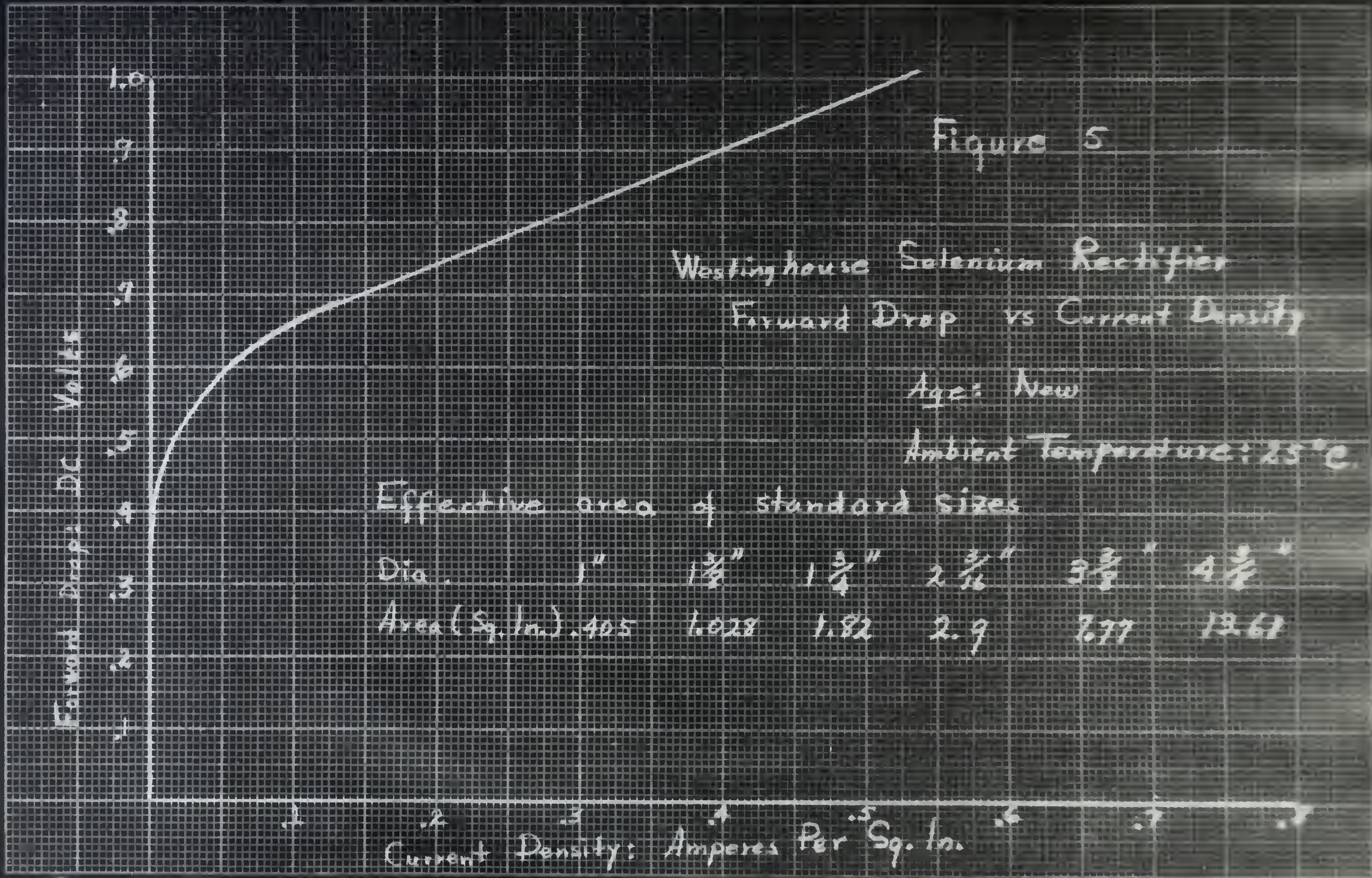
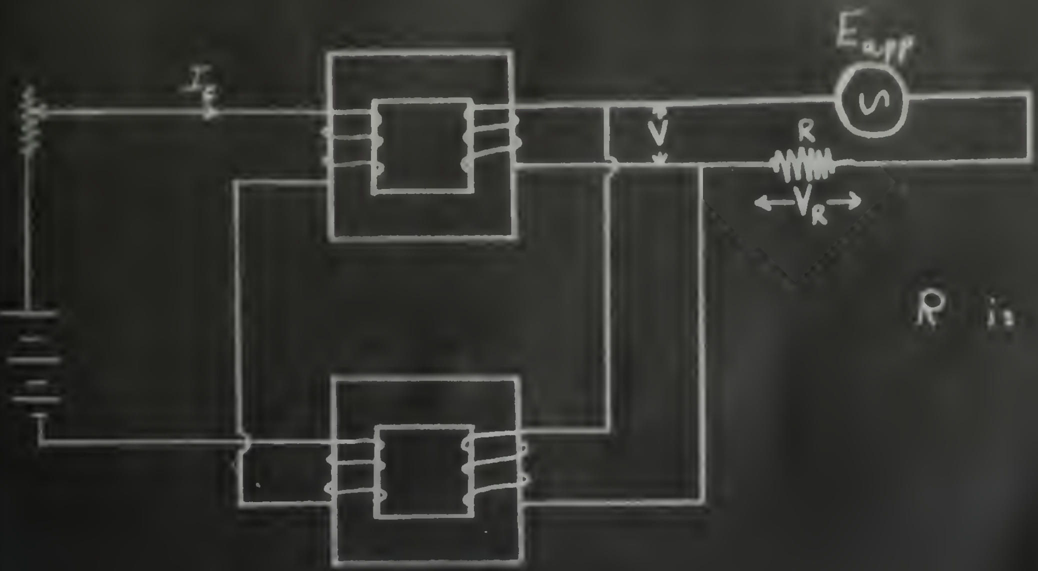


Figure 5
Westinghouse Selenium Rectifier
Forward Drop vs Current Density
Age: New
Ambient Temperature: 25°C.
Effective area of standard sizes
Dia. 1" 1 3/8" 1 3/4" 2 5/16" 3 3/8" 4 3/8"
Area (Sq. In.) .405 1.028 1.82 2.9 8.77 12.61
Forward Drop: DC Volts
Current Density: Amperes Per Sq. In.
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.1
.2
.3
.4
.5
.6
.7
.8



R is small – 5 or 6 Ω

Figure 6 – Circuit Used to Get H vs B Curves

$$L = NA \frac{dB}{di} \times 10^{-8}$$

$$= NA\mu \frac{dH}{di} \times 10^{-8}$$

$$= 1.256 \frac{N^2 A\mu}{l} \frac{di}{di} \times 10^{-8} \qquad (11)$$

Therefore, $$X_L = 7.89 \frac{f\mu AN^2}{l} \times 10^{-8} \qquad (12)$$

and $$I_{ac} = \frac{V_{ac}\, l \times 10^{8}}{7.89\, f\mu AN^2} \qquad (13)$$

$$\frac{\mu N I_{ac}}{l} = \frac{V_{ac} \times 10^{8}}{7.89\, fAN}$$

$$B_{ac} = \frac{12.68\, V_{ac} \times 10^{6}}{fAN} \text{ gausses}$$

In the above equations, V_{ac} is volts, I_{ac} is amperes, L is henries, but all other units are cgs. The Westinghouse Corporation, which produces hipersil, uses gausses, ampere turns per inch for H, square inches for A. Using these units,

$$L = 3.19 \frac{N^2 A\mu}{l} \times 10^{-8} \qquad (14)$$

$\mu = .313\, B/H$ where B and H are in English units.

Therefore, $$L = \frac{N^2 A\, B/H}{l} \times 10^{-8}$$

and $$X_L = 6.28 f \frac{AN^2\, B/H \times 10^{-8}}{l}$$

and $$I_{ac} = \frac{V_{ac}\, l \times 10^{8}}{6.28\, fAN^2\, B/H}$$

and $$B = \frac{V_{ac} \times 10^{8}}{6.28\, fAN} \text{ gausses}$$

But A must be converted to square inches.

$$\text{Therefore,} \quad B = 2.47 \frac{V_{ac} \times 10^6}{fAN}$$

This B is an "rms" flux density whereas the peak value causes the magnetic amplifier to fire. Therefore,

$$\hat{B} = 3.50 \frac{V_{ac} \times 10^6}{fAN} \qquad (15)$$

Where V_{ac} is rms volts across one reactor.

H in ampere turns per inch is determined by measuring the voltage across the resistor R, finding the current as $\frac{V_R}{R}$ and computing $\frac{NI}{l}$. Since this current is the current from two reactors, it is twice the value of one.

To get the $\hat{B}$-H curves, I_c is set at zero and V_{app} is increased from zero in convenient steps, and $\hat{B}$ and H are computed at each point until saturation has been passed. The locus of these points is a B-H curve for $I_c = 0$. I_c is now increased to, say, twenty milliamperes, and a new curve is obtained. This process is repeated at regular intervals until a family of curves such as are shown in Figure 7 is obtained. In these curves it may be noted that the H_{dc} exceeds the H_{ac} for one reactor by a small amount at and beyond the steepest part of each curve. (To get H_{ac} for one reactor, the values shown on the abscissa must be divided by two). The reason for this excess of H_{dc} over H_{ac} is that the total d-c magnetomotive force must exceed the a-c magnetomotive force by the amount required to drive the core into saturation.

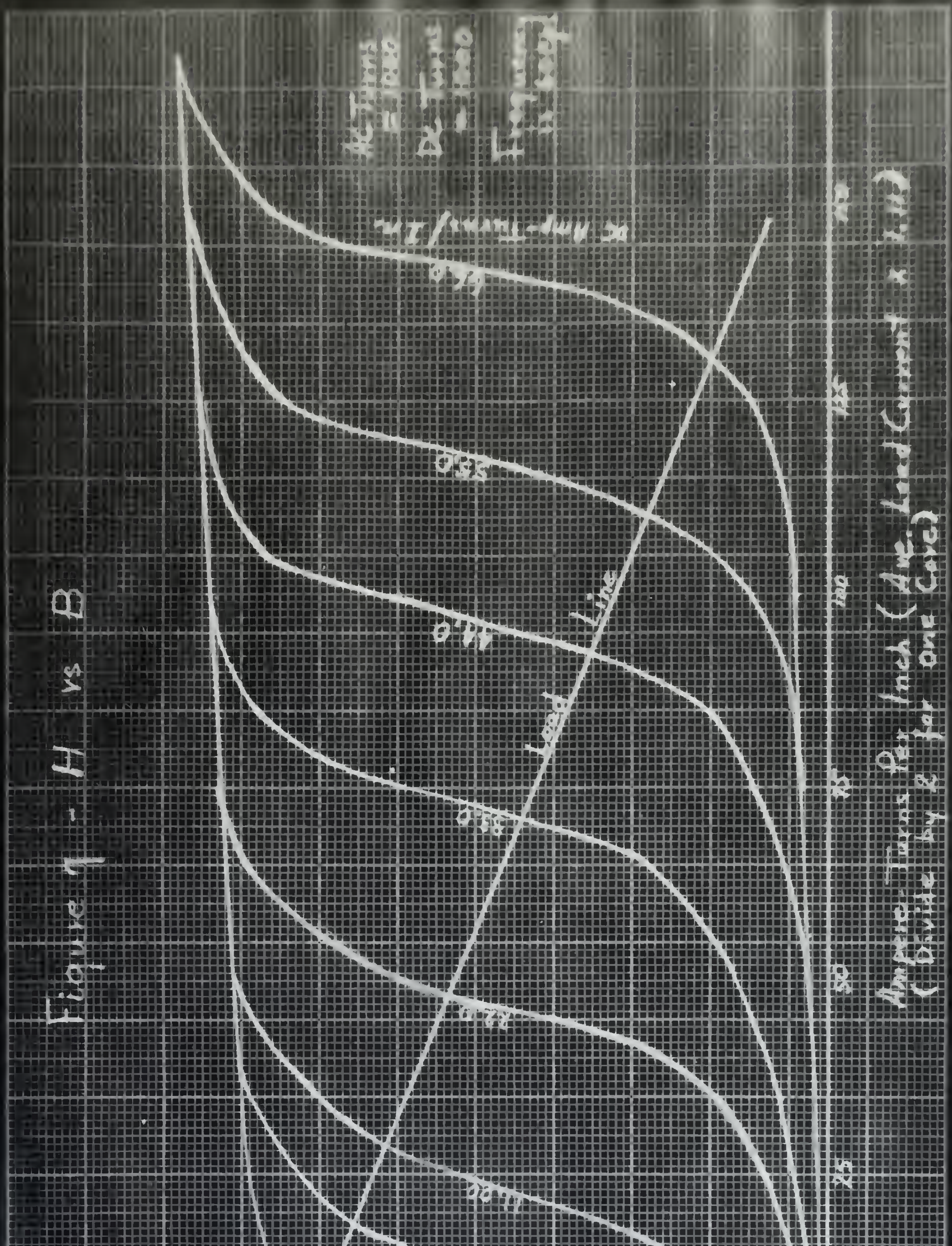


Figure 1 - H vs B
Load Line
Ampere-Turns Per Inch (Ave. Load Current x ...)
(Divide by 2 for one curve)

With the curves of Figure 7 available and with the knowledge f load conditions, the design of the magnetic amplifier may be accomplished. These data are known:

1. Maximum load conditions. This was previously determined from the experimental test on the restifier and load. Maximum load is a V_{max} and an I_{max}.

2. Minimum load condition. This was similarly determined from the restifier-load test.

Minimum load is a V_{min} and an I_{min}.

3. The length and area of the core.

The ratio

$$\frac{Max\ \left(\frac{NI}{l}\right)_{ac}}{Min\ \left(\frac{NI}{l}\right)_{ac}} \tag{16}$$

is now determined. The ratio (16) will be a numeric since N is the only unknown.

Choose a point near the knee of the I_c equals zero curve. This point will determine Min. $\left(\frac{NI}{l}\right)_{ac}$, which in turn determines Max. $\left(\frac{NI}{l}\right)_{ac}$ since the ratio of $H_{ac\,max}$ to $H_{ac\,min}$ is known. The point chosen near the knee of the I_c equals zero curve also fixes a maximum voltage across the reactor, since, as was shown in equation (15), $\hat{B} = 3.50 \frac{V \times 10^6}{fAN}$ gausses. This maximum reactor voltage occurs, of course, when the load voltage is a minimum/ Since the ordinate in Figure 7 is V/AN rather than B, V is easily obtained by multiplying the value of V/AN found opposite the point chosen near the knee by AN. Now the reactor voltage and load voltage are both known for minimum load. The applied voltage will usually be the quadrature sum of these. In some applications, however, the reactor

voltage, and the load voltage add almost arithmetically to give the applied voltage. The angle between these two voltages may be determined experimentally. The minimum voltage across the reactor may be found by subtracting the maximum load voltage from the applied voltage.

Thus two points are found on Figure 7--the first selected near the knee of the I_c equals zero curve, and the second determined by the intersection of minimum V and maximum $\left(\frac{NI}{\ell}\right)_{ac}$. A line drawn between these two points may be called the load line, and it represents the approximate locus of operation of the magnetic amplifier as is increased from zero to the value necessary to move the locus to the end point. If this load line does not fall within a useable range of the curves of Figure 7--for example, if V_{min} falls below the bottom envelope of the curves--, it is necessary to move the initial point either up or down on the I_c equals zero curve to a point which does cause the second point to fall on a practicable spot. The load line will not be straight in most applications. If the load voltage and reactor voltage add in quadrature, the locus will be elliptical. However, in many applications the best the designer can do is fix the end points. If, for the sake of linearity, it is essential to know the exact shape of the operating locus, intermediate points may be found most easily by experimental test.

N, the number of turns on the load winding of one reactor, may be readily found since both $\left(\frac{NI}{\ell}\right)_{ac\ min}$ and $\left(\frac{NI}{\ell}\right)_{ac\ max}$ are known, N being the only unknown in either. The wire size for the load windings is fixed by the I in $\left(\frac{NI}{\ell}\right)_{ac\ max}$. The wire is then wound around micarta tubing and fitted onto the core. Window space for the control winding plus any other desired windings must remain.

From Figure 7, $\left(\frac{NI}{\ell}\right)_{dc\ max}$ may be found as the line which passes through the end point of the operating locus. An N_{dc} is selected. N_{dc} should be at least twice as great as N_{ac} to reduce the control current. Once N_{dc} is selected, $I_{c\ max}$ is determined.

It is possible that the I^2R losses in the windings may cause excessive heat under maximum load. These losses may be determined by computing the resistance of the windings and then the total copper losses. Other losses may be neglected. Dividing the total copper loss per reactor by the total weights of the reactor, including windings, gives a figure in watts per pound that is a measure of whether or not overheating will occur. In the open air, a figure of one watt per pound will barely raise the reactor temperature by an appreciable amount.

An analysis of Figure 7 helps to clarify magnetic amplifier principles. Equation (14) indicates that the inductance of a load winding is equal to a constant times the core permeability. Or the inductance appears to equal a constant times the slope of the B-H curve at the operating point. A glance at the slopes of the curves of Figure 7 at the operating points proves that the "impedance" of the reactors certainly is not a direct function of the operating point slopes. In fact, except for the magnetizing current that flows under conditions of very low flux density, the currents that flow are not sinusoidal. Therefore, impedance is not simply a function of an inductive reactance, for there is no such thing as inductive reactance if the current is distorted unless the various harmonics of the total current are considered one at a time. Boyajian * showed

* Boyajian [1]

that if the saturation curve were divided into two regions, each being linear, the first region being that of unsaturation, the second that of saturation, the n^{th} harmonic for one reactor could be expressed by

$$i_n = \frac{2}{\pi}\left[\int_{\theta_1}^{\theta_2} i_1 \cos n\theta \, d\theta + \int_{\theta_2}^{\pi+\theta_1} i_2 \cos n\theta \, d\theta\right] \tag{17}$$

$$\text{where } i_1 = \frac{E_{max}}{X_1} \sin\left(\theta - \frac{\pi}{2}\right) \tag{18}$$

and

$$i_2 = \frac{E_{max}}{X_2} \sin\left(\theta - \frac{\pi}{2}\right) \pm i_k\left(\frac{X_1}{X_2} - 1\right) \tag{19}$$

Figure 8 (a) shows the saturation curve, 8(b) shows the component curves that add to give the resultant current shown in 8(c). The transients of regions one and two are represented by the second term of equation(19). i_k is the value of current represented by the knee of the saturation curve. This method of determining the various harmonic currents depends for its accuracy upon using high permeability cores. When I_c is large enough to move the operation point close to the full-load point, it becomes necessary to add a third region to the problem--that region extending almost horizontally from zero to the point at which the curve breaks upward.

As has been indicated, a general inductive reactance function does not exist when the load current becomes distorted due to saturation. Nevertheless, many references show a curve of X_L vs I_c, with the

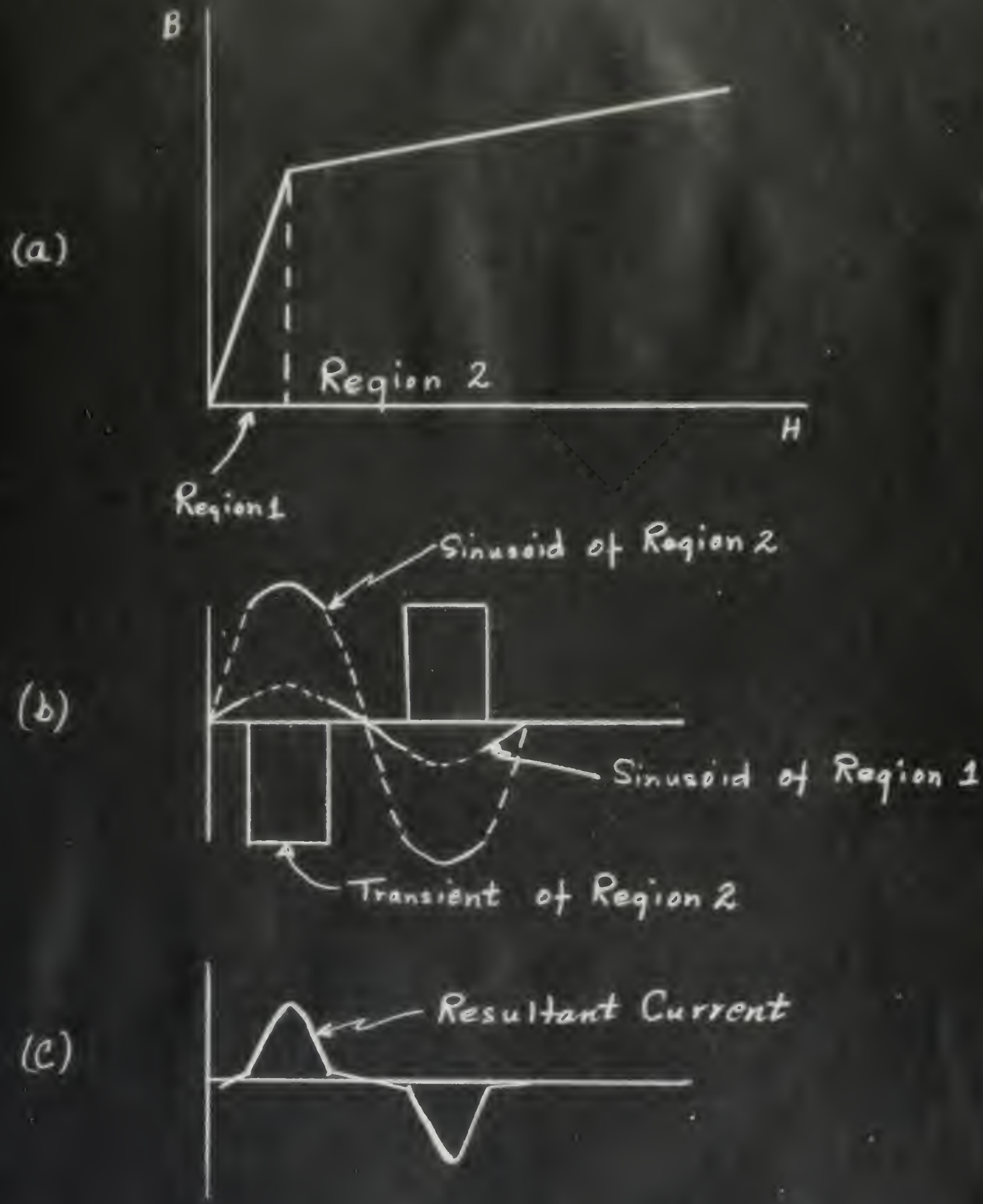


Figure 8 - Current Resulting from Simplified Saturation Curve.

decreasing exponentially from a high value for I_c equal to zero to a low value for large I_c. Such a curve is usually quite misleading, for it is usually constructed by entering curves like those of Figure 7 at points of constant flux density and varying control current. Since B is constant, V is constant and may be determined. Then

$$Z_{\text{one reactor}} = \frac{V}{I_{\text{one reactor}}} \tag{20}$$

$$= \frac{B \times \dfrac{fAN}{3.50 \times 10^6}}{H \times \dfrac{l}{2N}} \tag{21}$$

where H is the abscissa value of the intersection of the constant B line and the particular I_c line.

Or

$$Z_{\text{one reactor}} = k_1 \frac{B}{H} \tag{22}$$

Or

$$Z_{\text{one reactor}} = k_2 \frac{1}{H} \tag{23}$$

since B is a constant. Knowing this impedance for each H and knowing the resistance permits the determination of X_L for each H. A plot is then made of X_L vs I_c. Such a plot has very little justification, for the operation of a magnetic amplifier is always accompanied

by large changes in B. The only way B could remain constant would be for the supply voltage to be changed very drastically as I_c is changed. A true and useful plot of an "X_L" (not equal to $2\pi fL$) vs I_c could be made by letting H and B vary with I_c as actually happens. Equation (22) could be used for points along the load line. But Equation (23) could not be used since in it B was assumed constant.

2. Self-saturated Magnetic Amplifier Design

Besides the load and control windings, a reactor may also have windings for bias and feed back. Figure 7 may be used to obtain another series of curves called the transfer curves. These are obtained by drawing the load line across Figure 7 as explained in part A. Each intersection of this load line with a constant $\left(\frac{NI}{l}\right)_{dc}$ curve determines a value of $\left(\frac{NI}{l}\right)_{dc}$ and also a value of $\left(\frac{NI}{l}\right)_{ac}$. The latter is found on the abscissa directly beneath the point. A plot of $\left(\frac{NI}{l}\right)_{ac}$ vs $\left(\frac{NI}{l}\right)_{dc}$ may then be made. This may easily be transformed into a plot of I_{ac} vs I_{dc} and finally a plot of rectified load current vs I_{dc} (or I_c) as shown in Figure 9. The alternating current flowing in the load windings is a fairly linear function of the direct current in the control winding over most of the curve. Or it may be said that H_{ac} is a linear function of H_{dc} . Now, if part of the rectified load current is sent into a separate feedback winding in such a direction as to add to H_{dc} of the control winding, some of the direct H needed to produce a particular load current may be thus provided. It is possible, in fact, to provide so much positive feedback that the additional direct H needed from the control winding

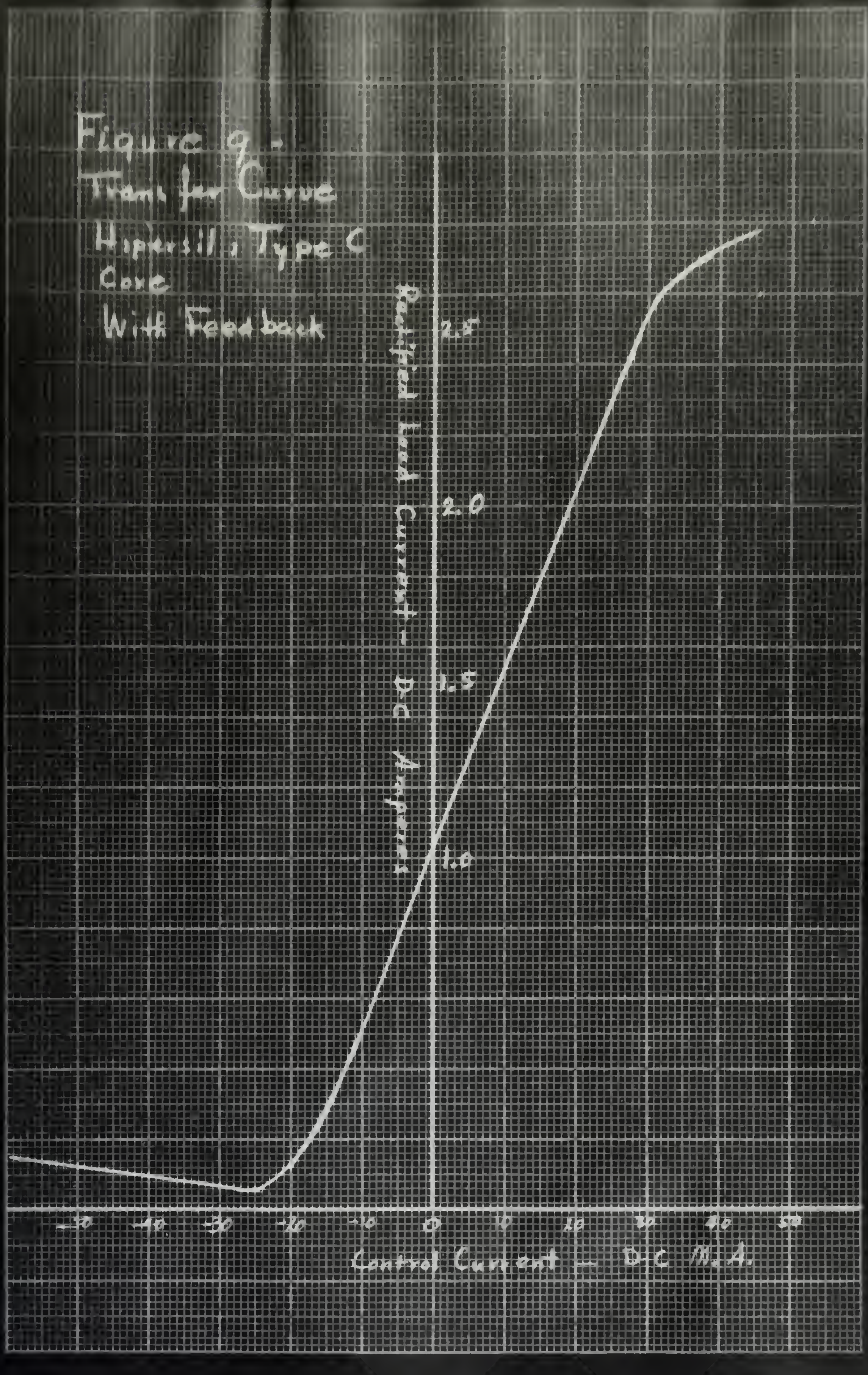


Figure 9 -
Trans fer Curve
Hipersil, Type C
Core
With Feedback
Rectified Load Current - D.C. Amperes
2.5
2.0
1.5
1.0
-50
-40
-30
-20
-10
0
10
20
30
40
50
Control Current — D.C. M.A.

may be only a small fractional part of that needed in the case where there is no feedback. This is illustrated in Figure 10. Figure 9 shows that maximum output with feedback may be obtained in a typical circuit with a control current of about forty milliamperes. Maximum output without feedback would require a control current many times this value, the I_c disparity depending upon the percent of feedback. The necessity of bias, in a separate bias winding, for certain applications is apparent in Figure 9. It is evident that for $\left(\frac{NI}{l}\right)_{dc}$ equal to zero, $\left(\frac{NI}{l}\right)_{ac}$ is not zero. If the bias current were of such a magnitude as to bring the minimum point of the curve far enough to the right to make it coincide with the $\left(\frac{NI}{l}\right)_{dc}$ equal zero ordinate, the a-c output for zero input would be materially reduced. This reduction of output for zero input might be essential in such applications as motor control if it is desired to be able to stop the motor.

The amount of feedback is

$$\beta = \frac{\left(\frac{NI}{l}\right)_{fdbk}}{\left(\frac{NI}{l}\right)_{ac}} \qquad (23)$$

Since l is the same for both windings,

$$\beta = \frac{(NI)_{fdbk}}{(NI)_{ac}} \qquad (24)$$

As has been pointed out, the total d-c *mmf* is the sum of the control current *mmf* and the feedback *mmf*. Or

$$(NI)'_{dc} = (NI)_c + \beta (NI)_{ac} \qquad (25)$$

Or

$$I_c = \frac{(NI)'_{dc}}{N_c} - \frac{\beta (NI)_{ac}}{N_c} \qquad (26)$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

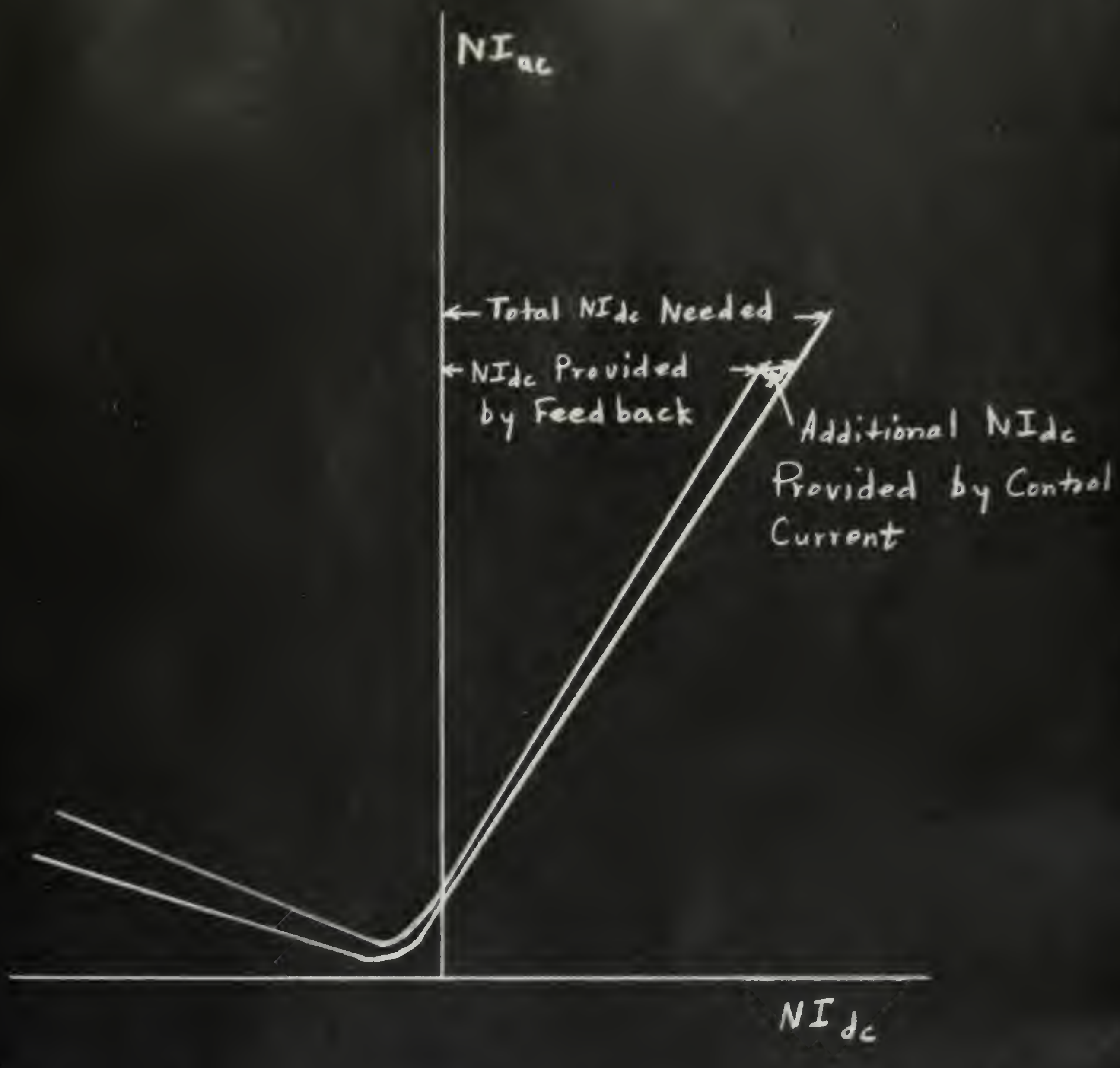


Figure 10 - NI_{ac} vs NI_{dc}

The extra winding for feedback may be eliminated by placing rectifiers in series with the load winding as shown in Figure 11. A direct component of the load current, proportional to the load current magnitude, provides the direct magnetomotive force formerly supplied by the feedback winding. This feedback arrangement is called self-saturation. Then N_{fdbk} and N_{ac} are one and the same winding and

$$\beta = \frac{I_{fdbk}}{I_{ac}} \tag{27}$$

also $$(NI)'_{dc} = (NI)_c + N_{ac} I_{fdbk} \tag{28}$$

so $$I_c = \frac{(NI)'_{dc}}{N_c} - \frac{N_{ac} I_{fdbk}}{N_c} \tag{29}$$

But $(NI)'_{dc}$ is also the magnetomotive force required with no feedback.

Therefore

$$(NI)'_{dc} = (NI)_{dc} \tag{30}$$

If it be assumed that N'_{dc} equals N_{dc}, then

$$I_c = I'_{dc} - I_{fdbk} \frac{N_{ac}}{N_c} \tag{31}$$

With two reactors in parallel as shown in Figure 11, I_{fdbk} is nearly equal to one-half of I_{load}. Since I_{load} may be measured, a more convenient expression for I_c is

$$I_c = I'_{dc} - \frac{I_{load}}{2} \frac{N_{ac}}{N_c} \tag{32}$$

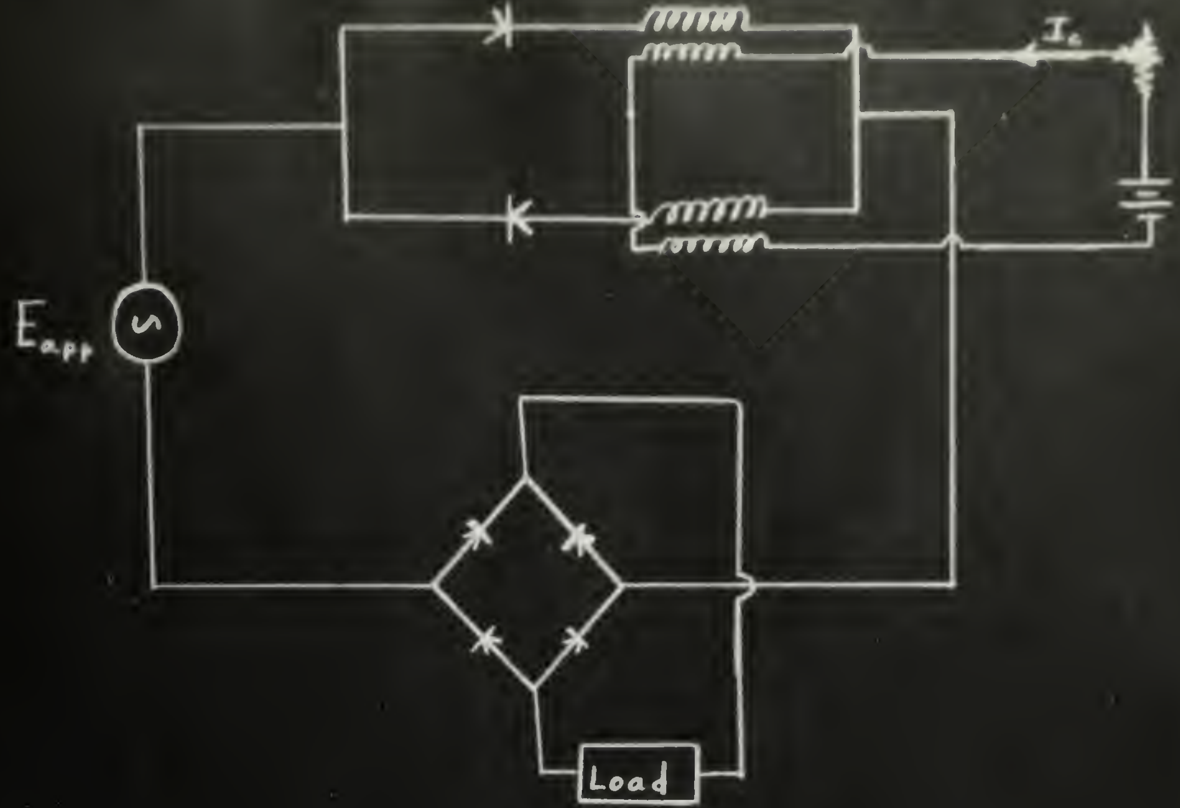


Figure 11 - Circuit Employing Self-Saturation

Self-saturation and external positive feedback are virtually identical. With self-saturation, reverse current through the rectifiers tends to demagnetize the core during the "non-conducting" half-cycle of the supply voltage. This demagnetization of the core may drastically reduce the amplification of the magnetic amplifier. It is important, therefore, to choose rectifiers having as great a back resistance as possible. With external feedback, there is a certain amount of flux leakage between the feedback winding and the load winding. This reduces the effectiveness of the feedback. Perhaps the principal consideration in contrasting the two methods of feedback is that of core utilization. It is clear that self-saturation, since it avoids the use of one winding, has the advantage of providing more space for other windings or permitting the use of a smaller core with the consequent economy of space and cost.

Since the input current (Ic) for a magnetic amplifier with feedback is much less than Ic for a magnetic amplifier without feeback, the power gain of the former is correspondingly greater. Whereas the power gain of a magnetic amplifier without feedback might be several hundred, the power gain of one with feedback might be as much as two hundred thousand.

As in the case of the simple magnetic amplifier of Section A, the firing time for the self-saturated amplifier can be controlled and determined. Before firing, the assumption is made that the entire supply voltage is across the reactors. After firing, it is assumed that the entire supply voltage is across the load. From

zero time, when the supply voltage is passing zero with positive slope, to some time later in the positive alternation all the supply voltage is across the reactors. When the flux density due to the voltage across the reactors reaches a critical value firing occurs, and the supply voltage shifts to the load. If now the control current is increased, the flux excursion in the reactors starts from a higher value and consequently reaches the critical (firing) flux density value earlier. This results in larger values of a-c magnetizing force and output voltage. B_o, the initial flux density, is determined by the control current only, since B_{ac} is zero when the supply voltage is zero. After the conducting half-cycle for one reactor, hysteresis and the presence of the rectifier causes the flux density to remain at a finite value even when I_c is zero. If hysteresis is appreciable, this value must be added to the flux density due to I_c to get B_o.

Since the voltage across one reactor is given by

$$e = N \frac{d\phi}{dt} \times 10^{-8} \qquad (33)$$

$$dB = \frac{e\,dt}{NA} \times 10^{8}$$

$$= \frac{E_m}{NA} \sin \omega t \, dt \times 10^{8} \qquad (34)$$

Therefore $$B = B_o + \int \frac{E_m}{NA} \sin \omega t \, dt \times 10^{8} \qquad (35)$$

where B_o is the constant of integration. Firing will occur when B exceeds the firing flux density B_f^*.

$$B_f = B_o + \int_0^{t_f} \frac{E_m \sin \omega t \, dt \times 10^8}{NA} \tag{36}$$

$$B_f - B_o = \frac{E_m \times 10^8}{\omega NA} \left(1 - \cos \omega t_f \right)$$

$$\cos \omega t_f = 1 + \frac{(B_o - B_f)\omega NA}{E_m \times 10^8}$$

$$\omega t_f = \cos^{-1}\left(1 - \frac{(B_f - B_o)\omega NA}{E_m \times 10^8} \right)$$

Let $\omega t_f = \theta_f$. Then, since

$$B_m = \frac{E_m \times 10^8}{\omega NA}, \tag{37}$$

$$\theta_f = \cos^{-1}\left(1 - \frac{B_f - B_o}{B_m} \right) \tag{38}$$

The limitations on equation (38) are: $-B_f \leq B_o \leq +B_f$

* Dornhoefer [2]

and $$0 \leq \theta_f \leq \pi \tag{40}$$

The average voltage across the load may now be computed by using

$$e_L = \frac{1}{2\pi}\int_{\theta_f}^{\pi} E_m \sin\theta \, d\theta$$

$$= \frac{E_m}{2\pi}(1 + \cos\theta_f) \tag{41}$$

or

$$e_L = \frac{E_m}{2\pi}\left(1 + \frac{B_m + B_o - B_f}{B_m}\right) \tag{42}$$

For two reactors, the output voltage given by equation (41) must be doubled. Since E_m is known and B_m is given by equation (37), the average load voltage may be determined if B_o and B_f can be found. If it is assumed that B_o is proportional to H_o only, where H_o is the mmf due to the control current alone, then B_o may be found by using

$$B_o = \mu H_o \tag{43}$$

where

$$H_o = \frac{NI_c}{l} \tag{44}$$

and μ is the slope, at the point corresponding to the given Ic, of the upper right branch of the hysteresis loop drawn for the core material with the a-c windings open. Or B_o may be read directly from the curve itself. B_f may be found from Figure 7 as the value of flux density at the knee of the curve drawn for the given Ic.

The foregoing method for finding the output of a self-saturated amplifier depends for its accuracy upon the validity of the assumptions made earlier. Its accuracy will be poor if before firing there is an appreciable voltage across the load and if, after firing, there is an appreciable voltage across the reactors.

3. Summary

The self-saturated magnetic amplifier circuit of Figure 11, is but one of many configurations that may be used. In Chapter II, for example, tests conducted on a bridge-type circuit will be discussed. Regardless of the circuit used, the principles of design outlined in this chapter may be applied if proper care is exercised. Cascaded magnetic amplifiers have not been mentioned for two reasons. First, cascading does not add to the design problem if special care is taken to ensure that there is no output direct current of the first stage (control current of the second stage) when the control current of the first stage is zero. This requires a balancing out of mmf between the two stages. Second, the special design problem, which is described in Chapter II, that was of particular interest did not require cascading. The design procedures detailed in this chapter are, in fact, the basis of the special design of Chapter II.

It may be recalled that the procedure of design did not specify what type of magnetic amplifier, simple or self-saturated, was to be used for a particular reactor design. As will be demonstrated in Chapter II, the reactors may be designed for a simple magnetic amplifier and then used in a self-saturated circuit under the same load conditions without modification of their physical make-up. It has been shown, however, that the self-saturated circuit is far superior to the type illustrated in Figure 1 in power gain. Therefore the self-saturated (or feedback) magnetic amplifier is almost always used in applications involving appreciable power consumption.

CHAPTER II

A MAGNETIC AMPLIFIER FOR SPS-6B RADAR ANTENNA CONTROL

1. The Problem

The problem was to design, build and test a magnetic amplifier to control the SPS-6B radar antenna. The requirements to be met were as follows:

(1) to rotate the antenna at any speed from zero to fifteen revolutions per minute under any load from zero to a sixty knot wind plus effect of inertial forces.

(2) to regulate the antenna's rotation speed as the load changed over any or all of the range from zero to a sixty knot wind plus effect of inertial forces.

(3) to eliminate vacuum tubes from the circuit.

It is felt the above requirements were met by the magnetic amplifier described in this chapter.

2. Antenna Motor Characteristics

(1) The motor was one-half horsepower, direct current, separately excited.

(2) The field required 115 volts, .12 amperes direct current.

(3) The armature voltage for full speed (3450 rpm) was 230 volts. d-c. For lower speeds, the armature voltage was, of course, less than 230 volts.

(4) The armature current varied with load as follows:

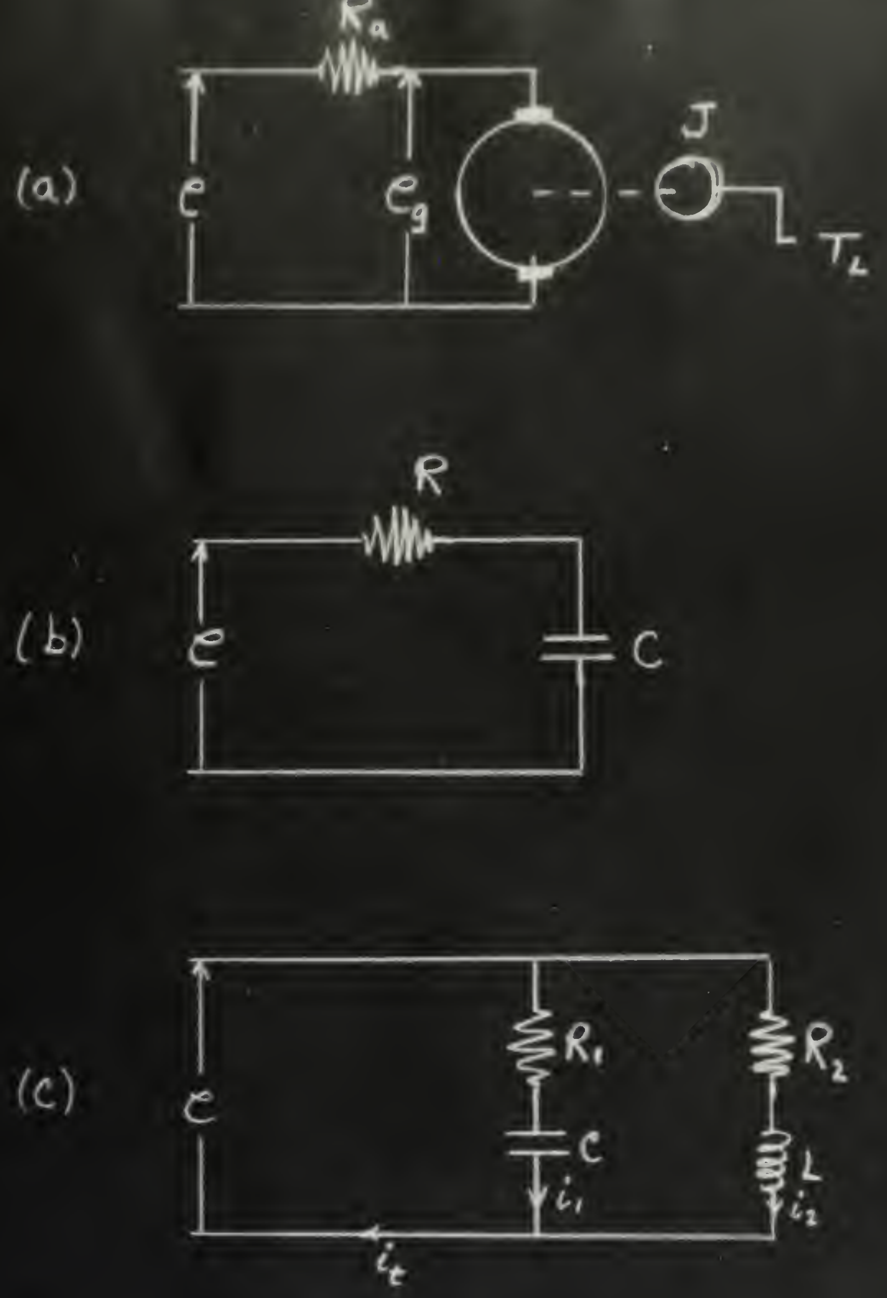


Figure 1 - Motor and Equivalent Circuits

No Load		Full Load	
RPM	I_a	RPM	I_a
creeping	.30 amps	creeping	.40 amps.
3450	.70 amps	3450	5.32 amps.

(5) The gear ratio from the motor to the gear extruding from the motor housing was 40:1. The ratio from this gear to the antenna was 5.75:1. Therefore the gear ratio from the motor to the antenna was 230:1 (or 3450:15).

The loading of the motor presented a problem since there was not available for testing purposes an SPS-6B antenna. A prony brake could have been used to simulate the antenna load, but such a method would involve errors. It was decided, therefore, to work out an electrical equivalent of the motor and load.* For the shunt d-c motor of Figure 1(a)

$$e_g = k_v \omega \tag{1}$$

and

$$t_m = k_T i \tag{2}$$

$$e = R_a i + e_g \tag{3}$$

or

$$e = R_a i + k_v \omega \tag{4}$$

The developed torque is

$$t_m = J \frac{d\omega}{dt} + T_L \tag{5}$$

or

$$k_t i = J \frac{d\omega}{dt} + T_L \tag{6}$$

Taking the Laplace transforms of equations (4) and (6), assuming initial conditions to be zero,

* From a paper by C. K. Hooper of the Westinghouse Corporation.

$$e(s) = R_a\, i(s) + k_v\, \omega(s) \tag{7}$$

$$k_T\, i(s) = J\, s\, \omega(s) + T_L(s) \tag{8}$$

Solving equation (8) for $\omega(s)$,

$$\omega(s) = \frac{k_T\, i(s) - T_L(s)}{J\, s} \tag{9}$$

Substituting $\omega(s)$ in equation (7),

$$e(s) = R_a\, i(s) + k_v \frac{k_T\, i(s) - T_L s}{J\, s} \tag{10}$$

or

$$e(s)\, J \cdot s = R_a\, i(s)\, J s + k_v k_T\, i(s) - k_v T_L(s) \tag{11}$$

or

$$e(s)\, J s + k_v T_L(s) = i(s)(R_a J s + k_v k_T) \tag{12}$$

or

$$i(s) = \frac{e(s)\, J s + k_v T_L(s)}{R_a J s + k_v k_T} \tag{13}$$

finally,

$$i(s) = \frac{\frac{J}{k_v k_T}\, s\, e(s) + \frac{T_L(s)}{k_T}}{R_a \frac{J}{k_v k_T}\, s + 1} \tag{14}$$

Both e and T_L may be called step functions of value $|e|$ and $|T_L|$ respectively. Then

$$i(s) = \frac{\frac{J}{k_v k_T}\, s\, |e| + \frac{|T_L|}{k_T}}{s \left[R_a \frac{J}{k_v k_T}\, s + 1 \right]} \tag{15}$$

If T_L is zero (load is zero),

$$i(s) = \frac{|e|}{R_a s + \frac{k_v k_T}{J}} \tag{16}$$

Now consider the circuit of Figure 1 (b). By Laplace,

$$e(s) = R\, i(s) + \frac{1}{C s}\, i(s) \tag{17}$$

or

$$i(s) = \frac{e(s)\, C s}{R C s + 1} \tag{18}$$

If again e is a step function of amplitude $|e|$,

$$i(s) = \frac{|e|}{Rs + \frac{1}{c}} \tag{19}$$

A comparison of equations (16) and (19) shows that a d-c motor at no load is equivalent to an R-C circuit. Further, it is seen that

$$R_a = R$$
$$\text{and } \frac{J}{k_v k_T} \text{ equals } C \tag{20}$$

The capacitance of the radar antenna motor at full speed was about 20,000 microfarads.

If T_L is not zero, the equivalent circuit for the motor under load follows from equation (15).

$$i(s) = \frac{|e|}{R_a s + \frac{k_v k_T}{J}} + \frac{|T_L|}{s\left(k_T + \frac{R_a J}{k_v} s\right)} \tag{21}$$

$$\text{or } i(s) = \frac{|e|}{R_a s + \frac{k_v k_T}{J}} + \frac{|e|}{s\left(\frac{|e|}{|T_L|} k_T + \frac{|e|}{|T_L|} \frac{R_a J}{k_v} s\right)} \tag{22}$$

Now consider the electrical circuit of Figure 1 (c). For the capacitive branch,

$$i_1(s) = \frac{e(s)}{R_1 + \frac{1}{cs}} \tag{23}$$

For the inductive branch,

$$i_2(s) = \frac{e(s)}{R_2 + Ls} \tag{24}$$

Adding,

$$i_t(s) = \frac{e(s)}{R_1 + \frac{1}{cs}} + \frac{e(s)}{R_2 + Ls} \tag{25}$$

If is a step function of magnitude $|e|$,

$$i_t(s) = \frac{|e|}{s(R_1 + \frac{1}{cs})} + \frac{|e|}{s(R_2 + Ls)} \qquad (26)$$

or $$i_t(s) = \frac{|e|}{R_1 s + \frac{1}{c}} + \frac{|e|}{s(R_2 + Ls)} \qquad (27)$$

A comparison of equations (22) and (27) follows in Table 1.

Table 1

Motor Symbol	Electric Circuit Symbol
R_a	R_1
$J/k_v k_T$	C
$\frac{\lvert e \rvert}{\lvert T_L \rvert} k_T$	R_2
$\frac{\lvert e \rvert}{\lvert T_L \rvert} \frac{R_a J}{k_v}$	L

The inductance term for the motor under test was about ten henries. However, the only effect this inductance will have on the d-c motor is to cause a momentary transient response for load or speed changes. Therefore, it is not necessary, in putting an equivalent load on the motor, to include this term. The equivalent load on the motor, then, is simply R_2. R_2 varies with the armature voltage and the real antenna load. For full load, V_a is 230 volts, horsepower is one-half, rpm is 3450, armature inertia J is .17 lb-ft².

Then $$I_a = \frac{\frac{1}{2}(746)}{230 \times .40} = 4.06 \text{ amps}. \qquad (28)$$

where the efficiency is .40. The full load torque is

$$T = \frac{HP \times 5252}{RPM} = .730 \text{ lb-ft} \qquad (29)$$

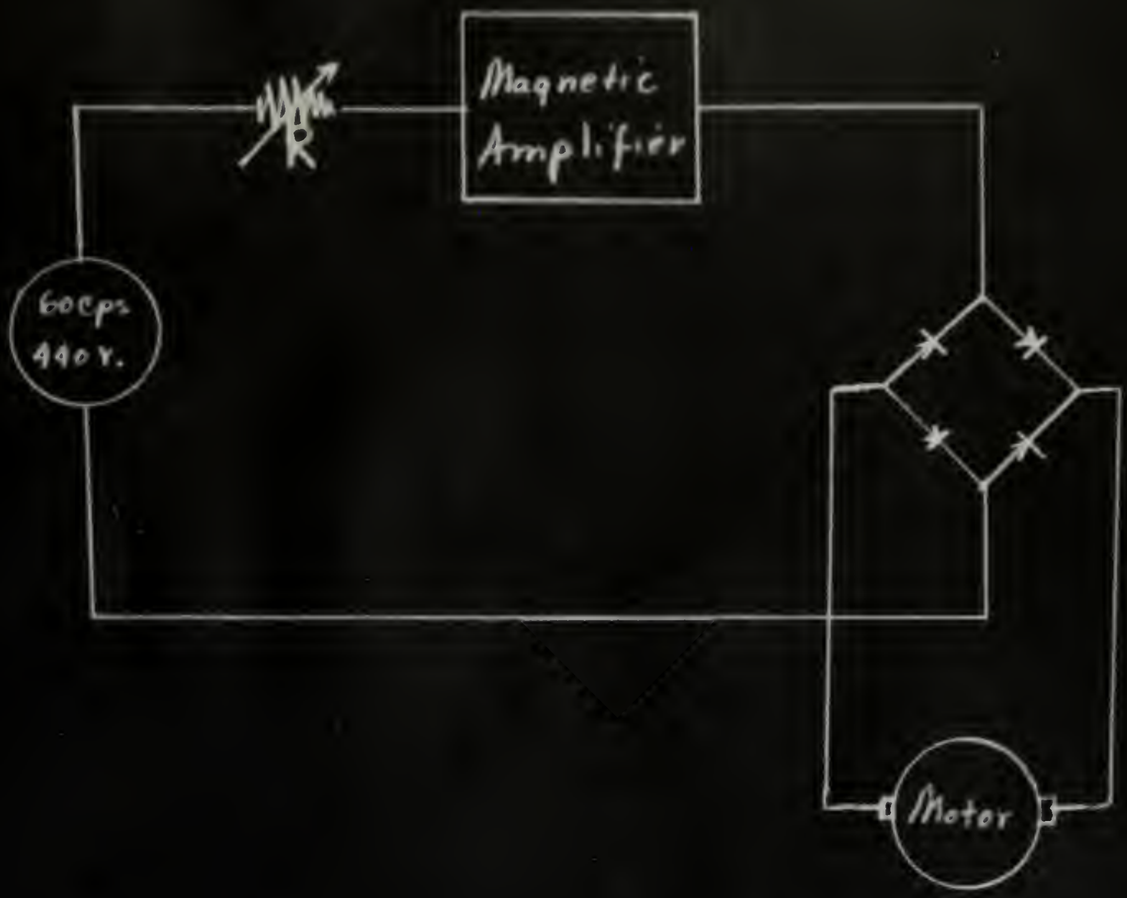


Figure 2 - General Method of Procedure

But $t_m = k_T i$. Therefore

$$k_T = \frac{.730}{4.06} = .18 \ \frac{\text{lb-ft.}}{\text{amp}} \qquad (30)$$

Therefore $$R_2 = \frac{230}{.76} \times .18 = 54.5 \ \Omega \qquad (31)$$

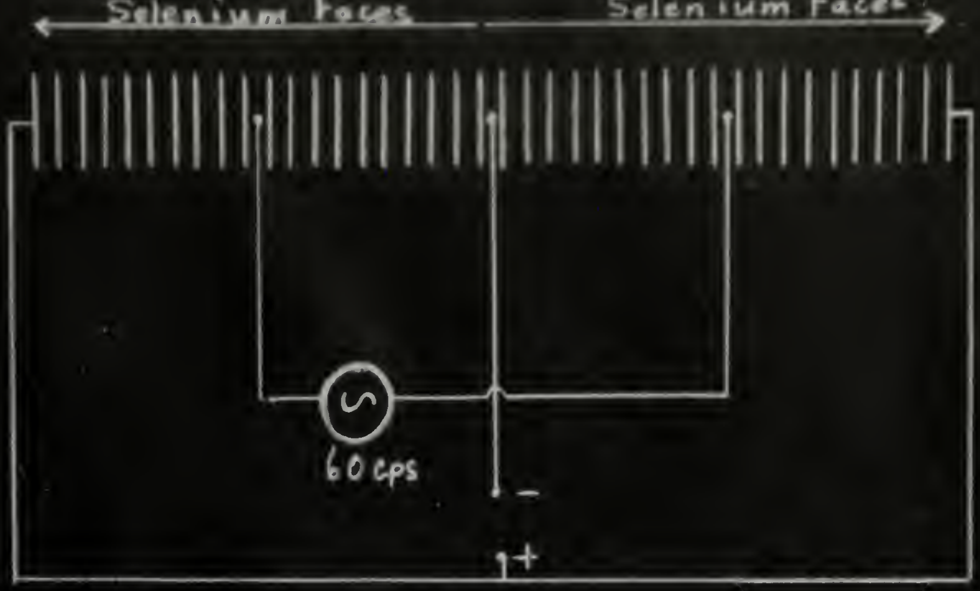
The resistor actually used in parallel with the armature to represent full load was fifty-five ohms. As will be shown, this value of resistance shunting the armature presented nearly the same load to the power source as the actual full load on the antenna (sixty knot wind plus effects of inertial forces).

Wind loading may result from ship roll, pitch or yaw as well as from the wind itself. In addition, roll, pitch or yaw throws an inertial load on the antenna due to its deviation from the balanced position. A third factor of rather minor consequence is the sinusoidal variation of load with antenna rotation. Balancing vanes on the antenna reduced the load variations resulting from the interaction of antenna rotary movement with ship movement and wind movement. It was found by test with the balanced antenna that a sixty knot wind resulted in a motor load of 4.6 amperes at full speed. Maximum mechanical loading added .72 amperes to this, giving 5.32 amperes d-c as full load, full speed armature current.

3. General Method of Procedure

The motor and the magnetic amplifier are connected in series with the 440 volt, 60 cycle supply as shown in Figure 2. By changing the impedance of the magnetic amplifier reactors, a varying d-c voltage is supplied to the motor armature. This variation must be such as to cause the motor to rotate at speeds from zero to 3450 rpm over the entire range of loads.

(a)



(b)

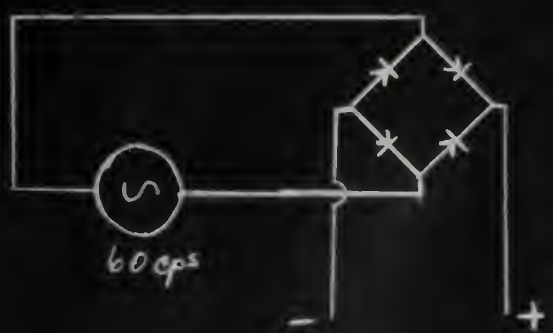


Figure 3 - Selenium Rectifier Arrangement

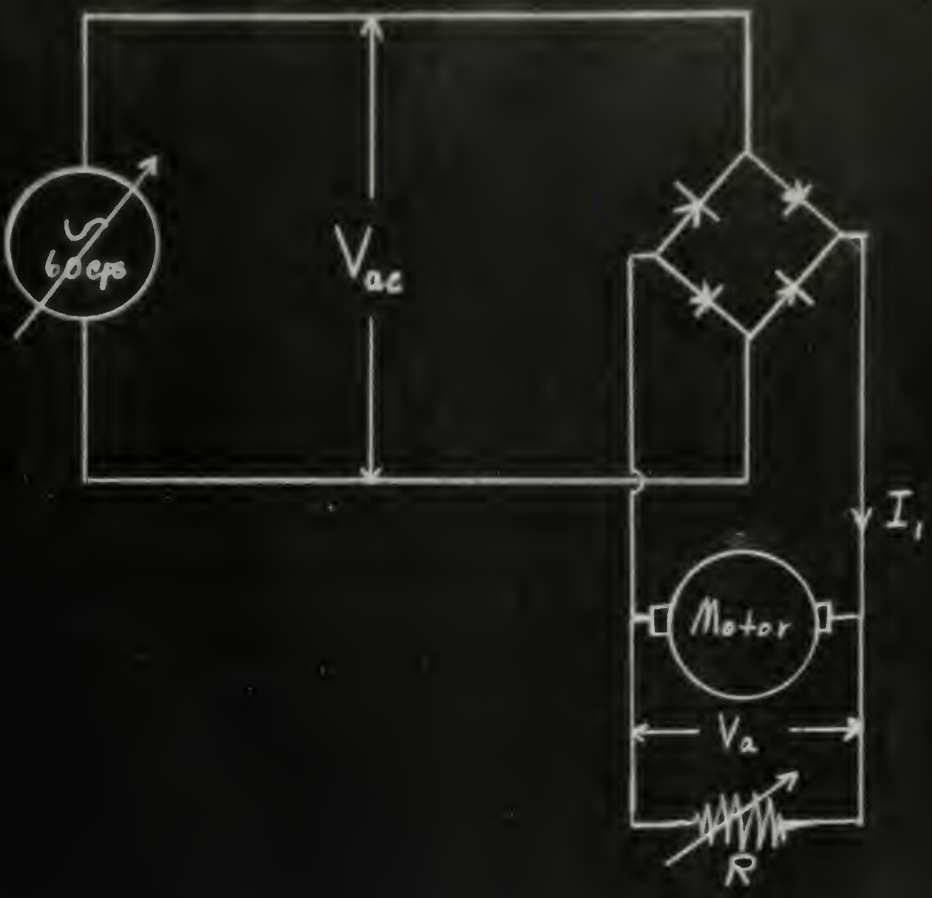


Figure 4 - Rectifier-Motor Test Circuit

4. The Design Data

Before the magnetic amplifier could be designed, the characteristics of the selected selenium rectifier over the load range had to be determined and considered in the problem. Figure 5 of Chapter I gives the forward voltage drop as a function of the current density for selenium rectifiers. It was considered advisable, however, to test the rectifier experimentally. A Westinghouse type "H" #6 selenium rectifier stack was selected, torn down and reassembled as shown in Figure 3(a). This is equivalent to the full-wave rectifier of Figure 3(b). In addition to this arrangement of rectifier plates, eighteen additional plates were used later in the self-saturated circuit.

To test the motor response when used with the rectifier, the circuit of Figure 4 was set up. The results are shown in Table 2 and Table 3 following:

Table 2

Vac	V_a	I_1	RPM	R
15 volts	2.2 volts	.26 amps	0	∞
15.9	4.0	.23	creeping	∞
22.4	10.0	.27	faster	∞

Table 3

Vac	V_a	I_1	RPM	R
15 volts	2.5 volts	.40 amps.	Creeping	55Ohms
23.8	10.0	.54	faster	55
57.0	40.0	1.27	faster	55
119.0	100.0	2.52	1400	55
176.0	150.0	3.58	-	55
214.0	180.0	4.22	2640	55
252.0	210.0	4.73	3260	55
276.0	230.0	5.10	3480	55

The data of Table 2 determine minimum load conditions. That is,

to stop the motor under zero load, the rms voltage applied to the rectifier must not exceed fifteen volts, and I_1 must not exceed about .25 amperes.

Table 3 shows that a resistor of fifty-five ohms shunting the armature did not quite give the same response as the actual wind and inertial loading. For the actual full load and full motor speed of 3450 rpm, I_1 was found to be 5.32 amperes. Table 2 gives a figure of 5.10 amperes for I_1 at full load and full speed. However, the discrepancy is slight, and "fullload" hereafter in this chapter will mean a resistance of fifty-five ohms shunting the armature.

The last entry in Table 3 above determines maximum load-maximum speed conditions. To get full speed under full load, 276 volts a-c had to be applied to the rectifier. 280 volts and 5.32 amperes were actually used in the calculations which follow.

Resume of preceding design data:

Minimum Load - Minimum Speed:

$V_{ac} = 15$ volts

$I_1 = .25$ amps

Maximum Load - Maximum Speed:

$V_{ac} = 280$ volts

$I_1 = 5.32$ amps

It is not necessary to consider "Minimum Load-Maximum Speed" or "Maximum Load-Minimum Speed", for these conditions fall within the range of operations encompassed by the two conditions outlined above.

5. The Magnetic Amplifier

The two cores selected were Westinghouse type "C" hipersil,

S#1366727 with the following dimensions per core:

Area: 3.08 sq. in.
L (mean core length): 14.85 in.
Window length: 4.1875 in.
Window width: 1.625 in.
Strip width: 2.1825 in.
Weight: 11.9 pounds
Volt-ampere capacity: 1450

The following design data were known:

Minimum Load-Minimum Speed:

$V_{ac} = 15\text{ v.}$

$I_1 = .278\text{ amps } (.25\text{ a.} \times 1.11)$

Maximum Load-Maximum Speed:

$V_{ac} = 280\text{ v}$

$I_1 = 5.90\text{ amps } (5.32\text{ a.} \times 1.11)$

The load current is multiplied by 1.11 because the characteristic curves of Figure 7 of Chapter I use this conversion factor.

The design data and the curves of Figure 7 of Chapter I were now used to design one reactor as follows:

$$\text{Min}\left(\frac{NI}{l}\right)_{ac} = \frac{.278}{14.85} N = .0187\, N \qquad (32)$$

$$\text{Max}\left(\frac{NI}{l}\right)_{ac} = \frac{5.9}{14.85} N = .397\, N \qquad (33)$$

$$\frac{\text{Min}\left(\frac{NI}{l}\right)_{ac}}{\text{Max}\left(\frac{NI}{l}\right)_{ac}} = .0471 \qquad (34)$$

and

$$N = \frac{\text{Min}\left(\frac{NI}{l}\right)_{ac}}{.0187} \qquad (35)$$

From curves of Figure 7 in Chapter I, a point on the I_c equals zero curve near the knee was chosen. Under this point, on the abscissa,

$$Min\left(\frac{NI}{l}\right)_{ac} = 7 \qquad (36)$$

and from equation (34),

$$Max\left(\frac{NI}{l}\right)_{ac} = 148.6 \qquad (37)$$

while from equation (35),

N = 376 turns per core.

From the curves,

$$\text{when} \quad Min\left(\frac{NI}{l}\right)_{ac} = 7$$

$$Max\ \frac{E}{AN} = .26 \qquad (38)$$

$$\text{and} \quad E_{max} = .26\ AN$$

$$= 302\ V.$$

Since the voltage across the rectifier was fifteen volts for this condition of Minimum Load - Minimum Speed,

$$V_{app} = 302 + 15 = 317\ V. \qquad (39)$$

This assumes that E and V should be added arithmetically. As was mentioned in Chapter I, usually E and V_{ac} should be added in quadrature, but it was also stated that there were exceptions. With the motor-rectifier load, the maximum angle between E and V_{ac} was about thirty degrees by test. Therefore the two voltages were added arithmetically.

$$\text{E for maximum load} = V_{app} - V_{ac}$$

$$= 317-280$$

$$= 37 \text{ volts}$$

$$\text{and} \left(\frac{E}{AN}\right)_{min} (\text{for max. load})$$

$$= \frac{37}{3.08 \times 376} = .033$$

Resume of preceding calcuations:

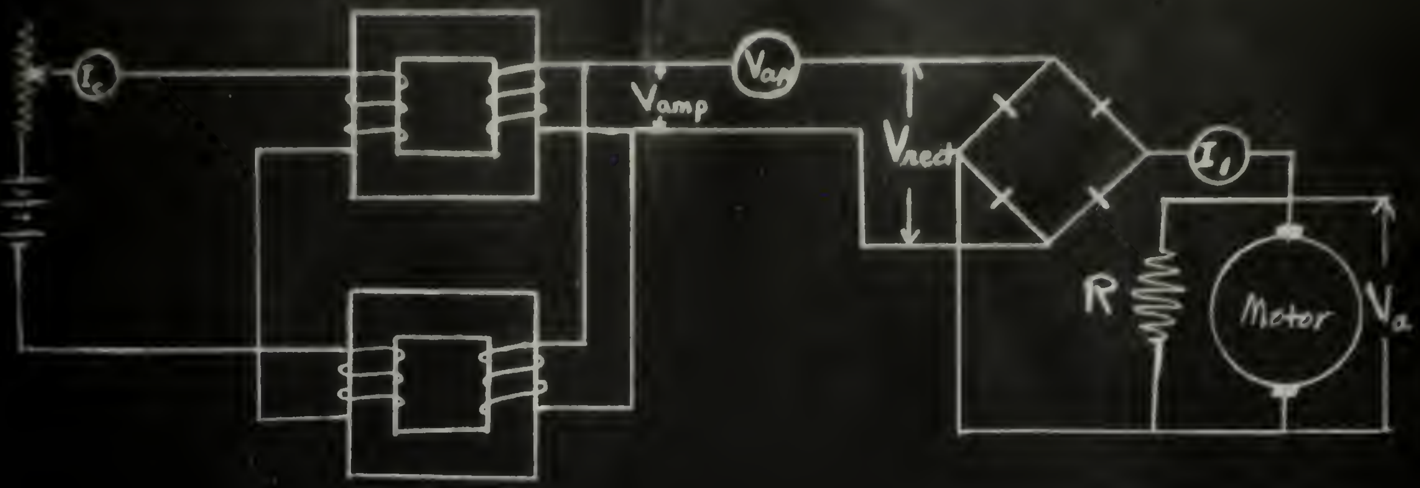


Figure 5 - First Magnetic Amplifier Circuit

Max. $\frac{E}{AN}$ = .26 for min. load and speed

Min $\frac{E}{AN}$ = .033 for max. load and speed

Min $\left(\frac{NI}{l}\right)_{ac}$ = 7.0 for min. load and speed

Max $\left(\frac{NI}{l}\right)_{ac}$ = 148.6 for max. load and speed

The a-c windings were made up of # 13 wire, 376 turns per core. From the curves of Figure 7 in Chapter I, the $\left(\frac{NI}{l}\right)_{dc}$ that passes through the full load, full speed point was eighty. N_{dc} was set at 760 turns per core. Then

$$I_{dc\,max} = \frac{14.85 \times 80}{760} = 1.56 \text{ amps} \qquad (40)$$

The wire selected was # 18.

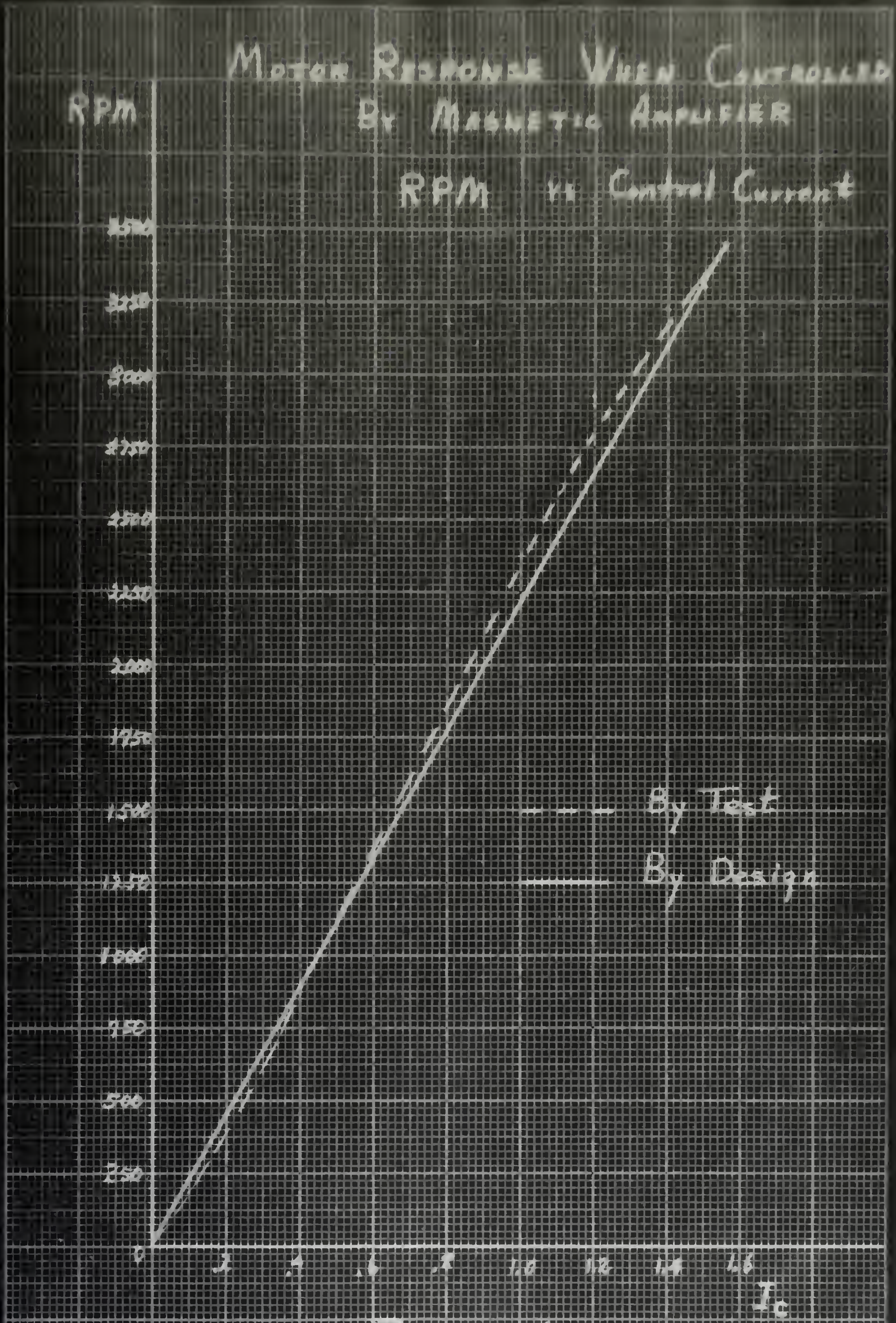
The copper loss per pound per reactor was calculated as follows:

Resistance of a-c coil = 1.0 ohm
Weight of a-c coil = 7.64 pounds
Resistance of d-c coil = 4.55 ohms
Weight of d-c coil = 3.53 pounds
Total weight of one reactor = 23.3 pounds
Max. d-c power loss = $1.55^2 \times 4.55$
= 10.2 watts
Max. a-c power loss = $2.95^2 \times 1$
= 8.7 watts
Total max. power loss = 18.9 watts
Copper loss per pound = $\frac{18.9}{23.3}$ = .811 watts per pound per reactor

Since a copper loss of one watt per pound will not cause appreciable heating, the calculated figure was on the conservative side.

6. Simple Magnetic Amplifier Circuit

The first circuit tested is shown in Figure 5. This circuit is the simple magnetic amplifier explained in Chapter I. The data of Table 4 and Table 5 were obtained.



Motor Response When Controlled By Magnetic Amplifier
RPM vs Control Current
RPM
Ic
By Test
By Design

Figure 6

Table 4

V_{app} = 300 volts R = ∞

I_c	Vamp	Vrect	RPM	I_1
.01 amps.	290 volts	16 volts	0	.30 amps
.03	283	32	143	.33
.10	250	100	910	.50
.20	100	280	3200	.64

The design data had V_{rect} equal to 15 volts when I_c was zero and the load and speed were minimum. This checks very well with Table 4.

Table 5

V_{app} = 310 volts R = 55 ohms

I_c	Vamp	Vrect	RPM	V_a
.025 amps	307 volts	16.0	creeping	-
.20	279	55.0	390	-
.40	248	98.0	930	-
.86	172	188.0	2000	135 volts
1.20	92	268.0	2900	190
1.50	55	295.0	3300	211

Note that the design data for full load gave full speed equal to 3450 rpm, V_a equal to 230 volts d-c, I_1 equal to 5.32 amperes, and I_c equal to 1.56 amperes. Increasing V_{app} and I_c to the design values would cause the tested motor speed, V_a and I_1 to approach very close to the design parameters. Figure 6 compares the actual motor response at full load with the response expected in accordance with the design. A feature noteworthy in this graph is the fair linearity of the motor response with control current. This could only mean that the load line drawn on Figure 7 of Chapter I was correctly drawn as a straight line rather than an ellipse.

The power gain of the magnetic amplifier was as follows:

$$\text{Gain} = \frac{\text{Power Output}}{\text{Power Input}} \tag{41}$$

(a) Minimum load - Maximum Speed:

Control circuit power $= I_c^2 R_c$

$= .2^2 \times 4.55$
$= .182$ watts

Load circuit power $= V_a \times I_a$
$= 210 \times .64$
$= 134.4$ watts

$$\text{Gain} = \frac{134.4}{.182} = 740$$

(b) Maximum load - maximum speed:

Control circuit power $= 1.50^2 \times 4.55$
$= 10.25$ watts

Load circuit power $= 211 \times 5$
$= 1055$ watts

$$\text{Gain} = \frac{1055}{10.25} = 103$$

As was shown in Chapter I, the angle during which one reactor fires is $180 - \theta_f$ where

$$\theta_f = \cos^{-1}\left(\frac{B_m - B_f + B_o}{B_m}\right)$$

Neglecting hysteresis, when I_c is zero B_o is zero. B_f on Figure 7 of Chapter I is represented by the knee of the saturation curve. Here B_f equals 16.4 kilogausses since

$$B = \frac{3.50 \times E \times 10^6}{f A N} \text{ gausses}$$

and $\frac{E}{AN} = .28$. Also B_m equals 15.2 Kg. Since $\frac{E}{AN}$ is .26. Therefore,

$$\theta_f > \cos^{-1}(0)$$

$$\text{or } \theta_f > 90^\circ$$

or each reactor is firing less than half the time for minimum load and minimum speed.

At full load and full speed, $\frac{E}{AN}$ is .033. Therefore B is 1.9 Kg.

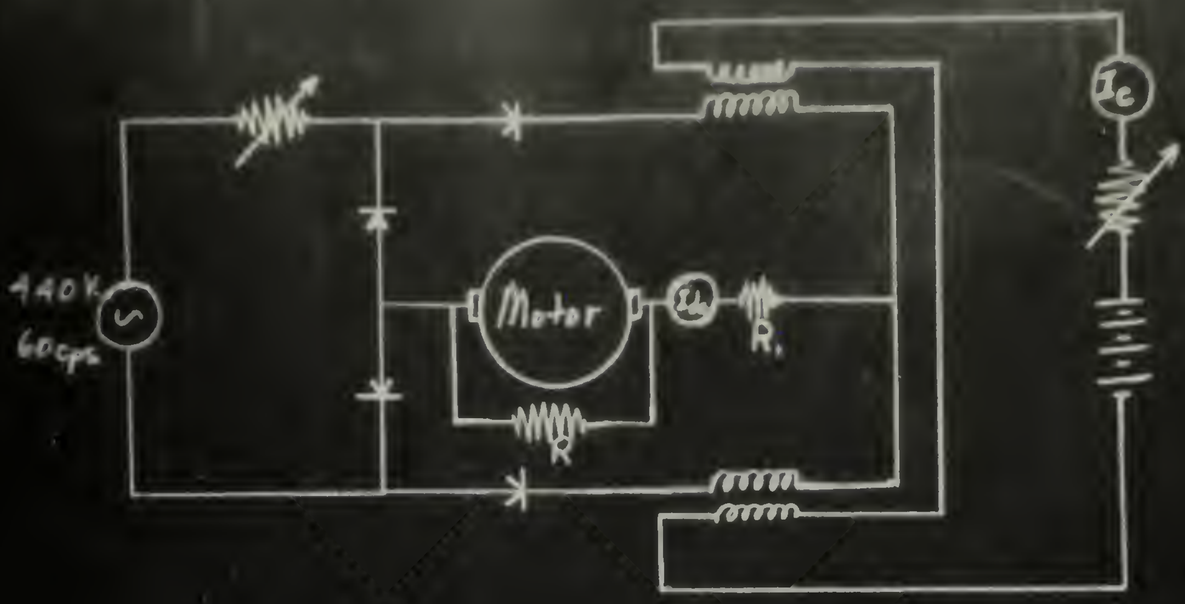


Figure 7 – First Circuit Using Self-saturation

B_f is now about 17.6 kg and B_0 is μH_0 or $B_0 = 175 \times 100 = 17.50$ kg.
Therefore

$$\theta_f = \cos^{-1}(1 - 9.28 + 9.22)$$
$$= \cos^{-1}(.94)$$
$$\theta_f = 20^\circ$$

or each reactor is firing for 160 degrees in its cycle. The firing times are not to be considered as rigorously correct since B_0 is difficult to determine accurately.

The magnetic amplifier operated satisfactorily. However, the necessity of having one and a half amperes of control current for full load-full speed made it desirable to design a self-saturated magnetic amplifier since such an amplifier needs much less control current.

7. Self-Saturating Magnetic Amplifier Circuits.

The self-saturating circuit of Figure 7 was set up. The feed-back magnetomotive force results from the direct current flowing through the load windings. As shown in Figure 10 of Chapter I, the control current required to give a particular degree of saturation may be greatly reduced by using this positive feedback.

Under full load, 3500 rpm was reached with V_{app} equal to 300 volts and I_c equal to 60 milliamperes. Therefore the sensitivity ratio of this circuit as compared with the simple magnetic amplifier is about $\frac{1.70}{60 \times 10}$ or 28.33, the ratio of the control currents required to reach the same motor speed. This assumes, by extrapolation, that 1.70 amperes of control current, with V_{app} equal to 300 volts, would have given a motor speed of 3500 rpm in the simple magnetic amplifier. This increase in sensitivity was gratifying. But the motor could not be stopped or slowed greatly with $V_{app} = 300$ volts no matter how

Figure 8 - Coil Arrangements Tried

the control windings and load windings were arranged. Figure 8 shows the arrangements tried. With zero control current, for example, the motor speed was 2300 rpm using one arrangement. No arrangement of the control windings as bias winding would reduce this speed much below 2000 rpm. This indicated that B_m for all values of B_o exceeded B_f and therefore firing could not be prevented. Since there are rectifiers in series with the reactors, B_o is not a function of the control current alone. To the flux density resulting from I_c must be added the residual flux density resulting from the direct current flowing through the load winding. As was shown in Chapter I,

$$B_m = \frac{B_f - B_o}{1 - \cos \theta_f} \qquad (42)$$

If θ_f is near zero, B_m increases to a large value unless B_o, which increases with B_m, approaches B_f. Then an indeterminate situation results. If B_o becomes greater than B_f, firing is, of course, continuous.

The power gain of this magnetic amplifier was as follows:

(a) Minimum load-maximum speed:

Control circuit power $= I_c^2 R_c$
$= .015^2 \times 4.55$
$= .00102$ watts

Load circuit power $= V_a \times I_a$
$= 230 \times .70$
$= 161$ watts

$$\text{Power gain} = \frac{161}{102 \times 10} = 158{,}000$$

(b) Maximum load-maximum speed:

Control circuit power $= I_c^2 R_c$
$= 36\text{x}10^{-4} \times 4.55$
$= .0164$ watts

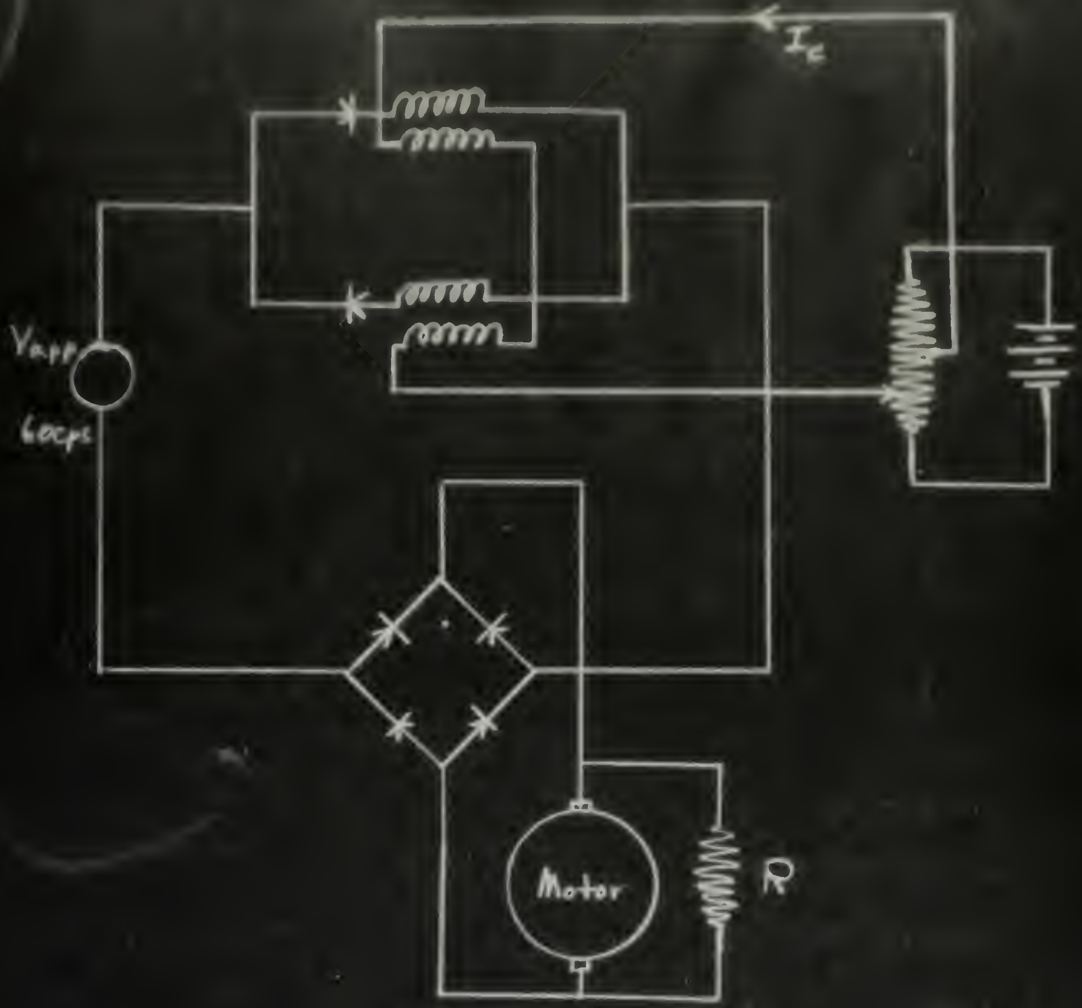


Figure 9 - Second Circuit Using Self-saturation

Load circuit power $= V_a \times I_a$
$= 230 \times 4.85$
$= 1115$ watts

Power gain $= \frac{1115}{.0164} = 68000$

Despite the excellent sensitivity of this circuit, it was abandoned because of its lack of controllability.

The self-saturation circuit of Figure 9 was set up. It was felt that this circuit would not be as sensitive as the first but would prove more controllable.

The following data were obtained from the new circuit:

(a) With V_{app} = 300 volts and full load:

I	RPM
0 ma.	1450
10	1900
20	2270
40	2700
60	3000
80	3200
100	3400

with control current reversed:

I	RPM
0 ma.	1450
10	1100
70	creeping

(b) With V_{app} = 300 volts and no load:

I	RPM
0 ma.	2700
10	3150
20	3450

with control current reversed:

I	RPM
0 ma.	2700
20	1600
40	600
70	creeping

Figure 10 - Typical Response Curve

If the reversed current in the control windings were increased beyond seventy milliamperes, the motor increased in speed. This is explained by a study of the response curve of Figure 10.

On the basis of the preceding data, V_{app} could be set at three hundred volts. Although the antenna could not then be stopped dead, its speed was perhaps one rpm in ten minutes, an impracticably slow speed. If, however, it were decided that it was desirable to stop the antenna dead on a target, either the armature voltage could be switched off automatically by the control current potentiometer when it was set to seventy milliamperes reversed control current, or the applied voltage V_{app} could be made 290 volts rather than 300. The latter arrangement will permit stopping the motor dead, but it will decrease the sensitivity since more control current will be required for full speed than was necessary when V_{app} was 300 volts.

Power gain of the magnetic amplifier was as follows:

(a) Minimum load-maximum speed:

$$\text{Control circuit power} = I_c^2 R_c = .02^2 \times 4.55 = 1.82 \times 10 \text{ watts}$$

$$\text{Load circuit power} = V_a \times I_a = 230 \times .7 = 161 \text{ watts}$$

$$\text{Power gain} = \frac{161}{1.82 \times 10} = 88{,}500$$

(b) Maximum load-maximum speed:

$$\text{Control circuit power} = I_c^2 R_c = .1^2 \times 4.55 = .0455 \text{ watts}$$

$$\text{Load circuit power} = V_a \times I_a = 230 \times 5.30 = 1222 \text{ watts}$$

$$\text{Power gain} = \frac{1222}{4.55 \times 10} = 26{,}800$$

In Chapter I, equation (32) gave an expression for the control current needed in a self-saturated magnetic amplifier. This equation was

$$I_c = I'_{dc} - \frac{1}{2} I_{load} \frac{N_{ac}}{N_c} \qquad (43)$$

where I_c is the control current in the self-saturated amplifier, I'_{dc} is the control current needed in a simple magnetic amplifier to give a particular output, and I_{load} is the average value of the load current. For full load and full speed I_c is 1.56 amperes, I_{load} is 5.32 amperes. Therefore

$$\begin{aligned} I_c &= I'_{dc} - \frac{1}{2}(2.66) \\ &= 230 \text{ ma.} \end{aligned} \qquad (44)$$

Actually, for full load and full speed I_c was 100 milliamperes. The disparity between the two figures for I_c is due to the fact that equation (43) is based on some assumptions that are legitimate enough but which make it give apparently erratic results because its result comes from the difference of two quantities that are close together. For a motor speed of 2400 rpm, I'_{dc} was 1.0 amperes, I_{load} was 3.97 amperes. Therefore

$$\begin{aligned} I_c &= 1 - \frac{1}{2}(1.985) \\ &= 7.5 \text{ ma.} \end{aligned} \qquad (45)$$

Actually, I_c for a motor speed of 2400 rpm was about thirty milliamperes in the self-saturated amplifier.

According to a formula developed in Chapter I, the output voltage (here across the rectifiers) due to both reactors is

$$e_L = \frac{E_m}{\pi}\left(2 + \frac{B_o - B_f}{B_m}\right) \qquad (46)$$

For maximum load and maximum speed, assume B_o equals B_f. Then

$$e_L = \frac{850}{\pi} = 270 \text{ v.}$$

The measured value was 295 volts. The difference between these two values may be due to one or both of the assumptions made being inadmissible. The first assumption was that the output due to the so-called pre-firing skirt was negligible, i.e., before firing all the supply voltage was across the reactors. However, if this assumption were wrong, the error would tend to increase the calculated output. Therefore for maximum load and speed, the assumption that B_o equals B_f may be incorrect. B_o must exceed B_f to make e_L approach more closely the measured output. If B_o were to exceed B_f, the amplifier would tend to fire on at least part of the negative alternation of the alternating flux. While this is possible, it would not add to the output because the rectifiers are unidirectional. Therefore, the condition previously made that $-B_f \leq B_o \leq +B_f$ need not be violated.

Of the three magnetic amplifiers tested, the last seemed to have the greatest potentiality. Therefore it was used in the remainder of the problem.

8. Speed Regulation

In the last circuit of Section 7 one hundred milliamperes of I were required to produce full speed at full load, but only twenty milliamperes of I_c are required to produce full speed at no load. This means that if the radar operator were to set the antenna speed control (I_c) at a certain point, the antenna speed would vary radically with changes in wind velocity, ship's speed, roll and pitch, and ship's heading. Therefore, the speed of the motor had to be

regulated so that a particular setting of I_c would give the same speed regardless of load.

Since it was known that a reversed current in the control winding slowed the motor, a "regulating" winding of two hundred twenty-five turns of #28 cotton-insulated wire was wound on each core. in the same sense as the control winding. A check was then made to see if reverse current in the regulating winding slowed the motor. With no load, V_{app} equal to 280 volts, and I_c equal to ten milliamperes, the following results were obtained:

I_d	RPM
1 ma.	2780
56 ma.	1700

Therefore this winding, with the regulating current, I_d , in the proper direction did slow the motor down. Similarly, if I_d had been reversed, the motor would have increased in speed. It was felt that the regulation should be degenerative rather than regenerative to minimize the possibility of instability. So the regulating current was made to slow the motor.

Equation (1) indicates that motor speed is directly proportional to the counter emf developed by the motor, or the armature voltage. If I_c were set at a particular value giving a certain motor speed and the load were to decrease, the armature voltage would increase and so would the speed. Now, if the increase in armature voltage were used to increase I_d , the motor speed would be decreased. The circuit of Figure 11 was set up to try this idea.

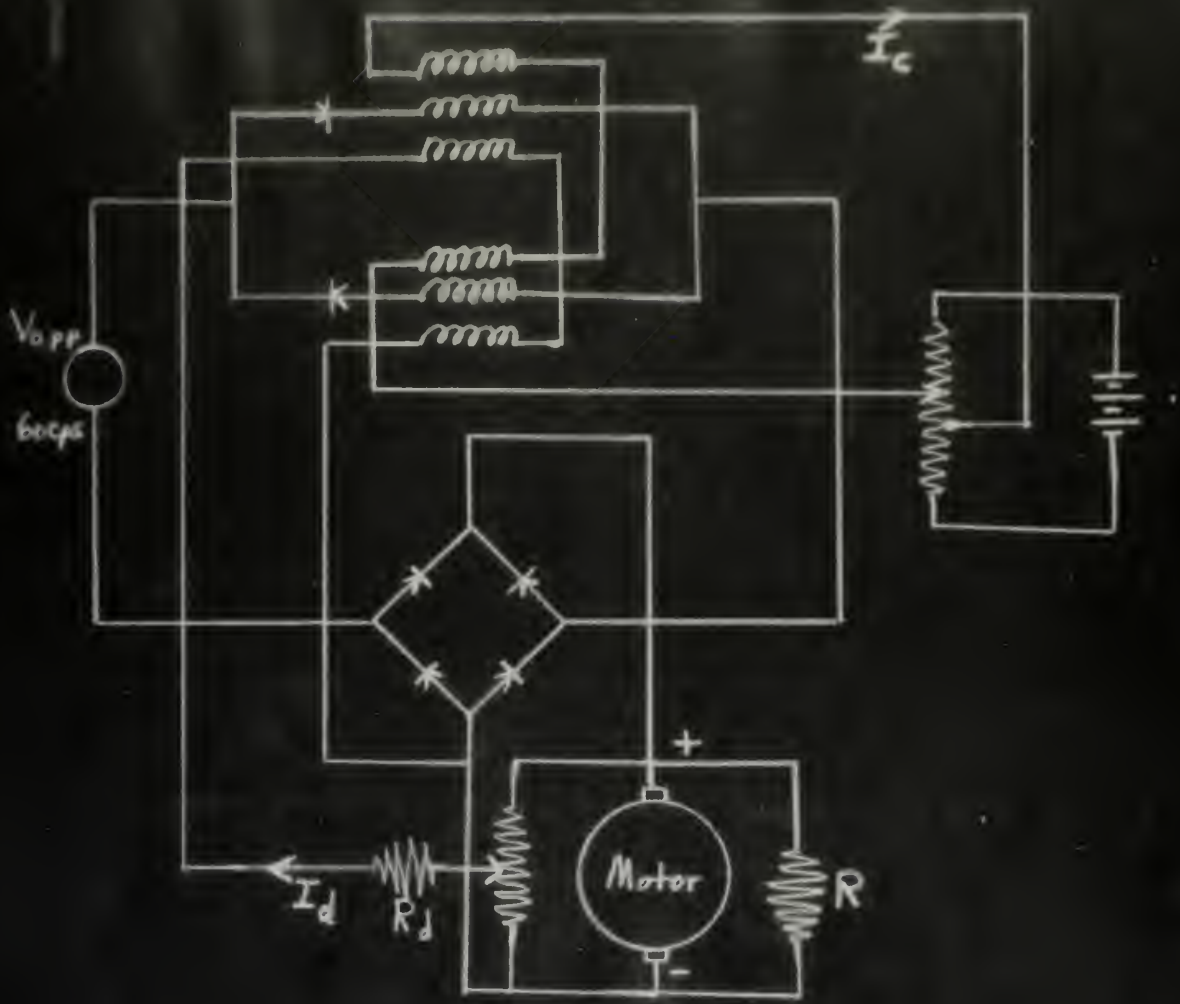


Figure 11 - First Regulating Circuit

In the circuit of Figure 11, regardless of load, V_a is fixed for a certain speed. For a particular I_c , motor speed (and V_a) depend upon load. For example, as the load decreases V_a increases, tending to increase the motor speed but also increasing I_d . This increase in I_d tends to slow the motor down. Of course, there will be some I_d for full load also, but it will increase as the load decreases.

Table 6 indicates how well this method regulated the motor speed.

Table 6

V_{app} =280 volts

R_d =500 ohms

Full Load			No Load		
I_c	I_d	RPM	I_c	I_d	RPM
10 ma.	57 ma.	880	10ma.	81 ma.	1180
20	72	1050	20	100	1300
40	100	1300	40	141	1610
60	138	1620	60	195	2000
80	175	1900	80	235	2350

With no regulation (I_d circuit open), the following data were obtained for comparison with the results shown in Table 6 for regulation:

Table 7

V_{app} = 290 volts

Full Load I_c	Full Load RPM	No Load I_c	No Load RPM
10 ma.	1660	6 ma.	2550
20	2030	10	3000
40	2510	20	3300
60	2800	25	3500
80	2940		

The graph of Figure 12 shows the improvement of regulation. With no regulation and I_c equal to twenty milliamperes, the motor speed difference for no load and full load was

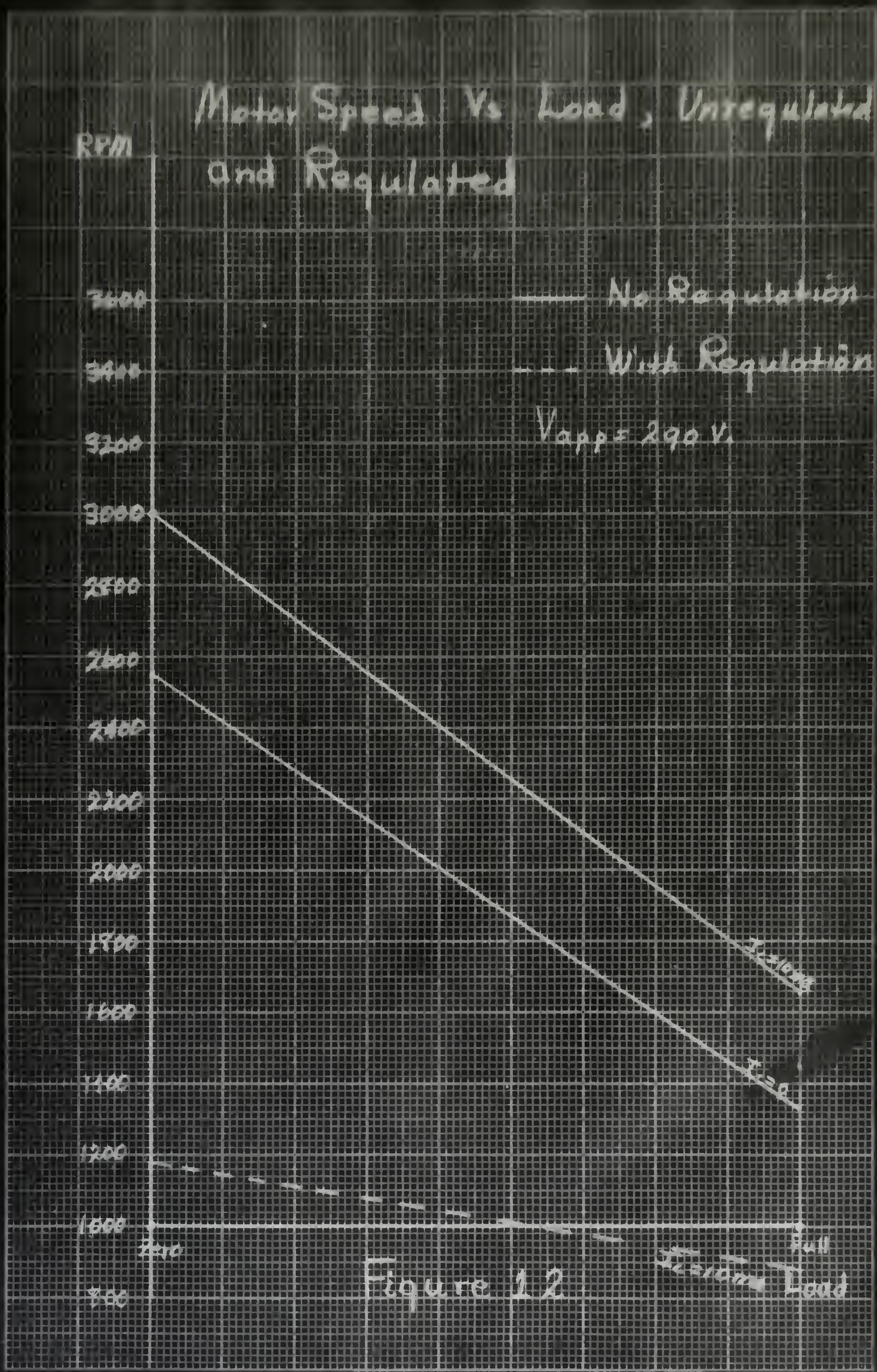
$$3300 - 2030 = 1270$$

With regulation and I_c equal to twenty milliamperes, the motor speed difference was

$$1300 - 1050 = 250$$

Two results may be noted. First, there was considerable improvement in regulation. Second, there was a material reduction in motor speed for all loads with regulation in effect. The speed, for a particular I_c , was approximately halved. It was considered desirable to modify this circuit in such a way as to decrease this sensitivity reduction. Figure 13 shows this modification.

In this circuit there will be no I_d at all for full load. This is achieved by setting P_2 so that V_d equals V_a for a particular setting of I_c corresponding to full load. If the load decreases, the motor speed and V_a tend to increase, causing I_d to flow to reduce the motor speed and V_a .


Motor Speed Vs Load, Unregulated
and Regulated
RPM
3600
3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
No Regulation
With Regulation
Vapp = 290 V.
Zero
Full
Load
Figure 12

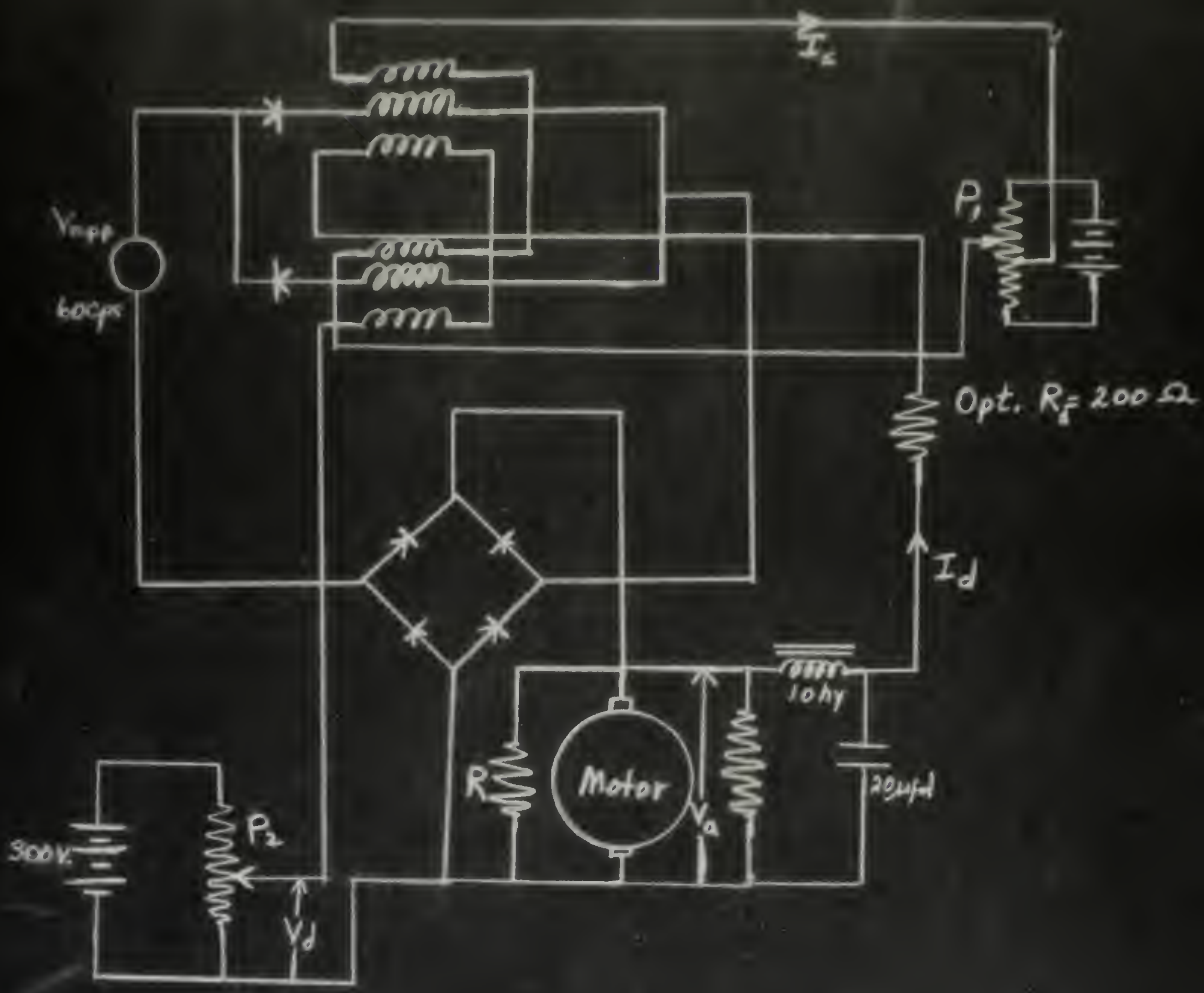


Figure 13 - Second Regulating Circuit

In actual operation, P_1 (the speed control) and P_2 (the regulation control) would be adjusted synchronously. In other words, P_2 would be so designed that its movable arm would be moved in synchronism with that of P_1 so that V_d would automatically be set equal to full load V_a for a particular I_c .

Once P_2 is set, it is necessary that V_d remain constant regardless of what V_a does. This poses a difficult problem. By making P_2 a very low resistance, of the order of one hundred ohms, V_d will remain constant as V_a changes, but the current drain and power wastage is exorbitant. To avoid this, it is necessary that the resistance of P_2 be large but that the resistance between the movable arm and common depend upon the magnitude of V_a . In other words, this resistance to common must decrease when V_a increases. This clearly indicates the need of a regulating device across V_d . A voltage regulating circuit such as shown in Figure 14 will vary the resistance from center tap to common in the way desired to keep V_d constant. An ordinary regulated power supply as the source of V_d is rapidly "unregulated" when V_a exceeds V_d .

Data on the regulating circuit of Figure 13 is given in Tables 8 and 9, which follow.

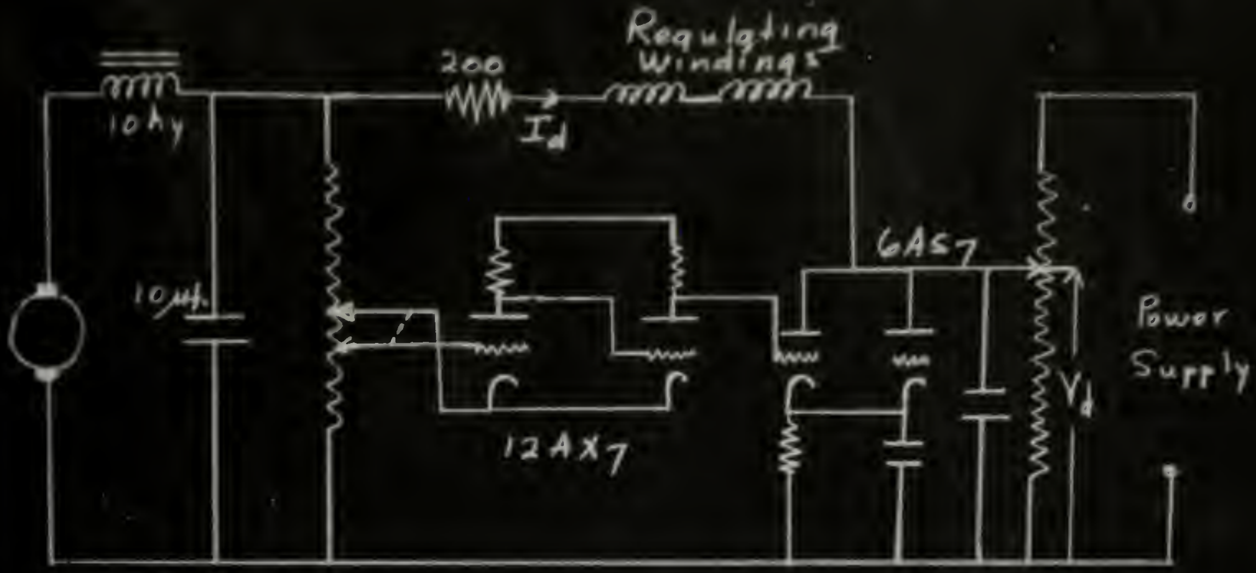


Figure 14 - Suggested Circuit to Hold V_d Constant

Table 8

V_{app} = 280 volts

R_d = 200 ohms

I_c = 0 ma.

	Load	RPM	V_a	I_d	V_d
Full Load	2.10 amps.	1200	80 v.	0	80 v.
	2.00	1200	82	3 ma.	80
	1.60	1200	84	16	80
	1.20	1270	86	21	80
	1.00	1300	88	28	80
Zero Load	.62	1340	92	53	81

Table 9

V_{app} = 280 volts

R_d = 200 ohms

I_c = 40 ma.

	Load	RPM	V_a	I_d	V_d
Full Load	3.90 amps.	2700	170 v.	0	169 v.
	3.60	2700	172	15 ma.	169
	3.20	2710	174	20	169
	2.80	2740	175	24	169
	2.40	2800	179	36	169
	2.00	2850	180	44	170
	1.60	2870	182	59	170
Zero Load	.72	2970	190	70	170

From Table 8 it is seen that with I_c equal to zero, if the load changed from full to no load, the antenna speed would increase by

$$\frac{1340-1200}{230} = .61 \text{ rpm}$$

where 230 is the gear ratio from motor to antenna. Since it is unlikely that the load would change rapidly from a sixty knot wind (plus roll and pitch of the ship) to a zero knot wind, this variation was considered negligible. It was better regulation than the electron tube regulating circuit presently used in the SPS-6 radar.

From Table 9, with I_c equal to forty miliamperes, when the load changed from full to no load, the antenna speed increased by

$$\frac{2970-2700}{230} = 1.23 \text{ rpm}$$

Again this change was considered negligible.

The graph of Figure 15 shows the improvement of regulation by this method.

As R_d was decreased, the incremental I_d increased for a given V_a change. However, as R_d is reduced it becomes more difficult to maintain V_d constant.

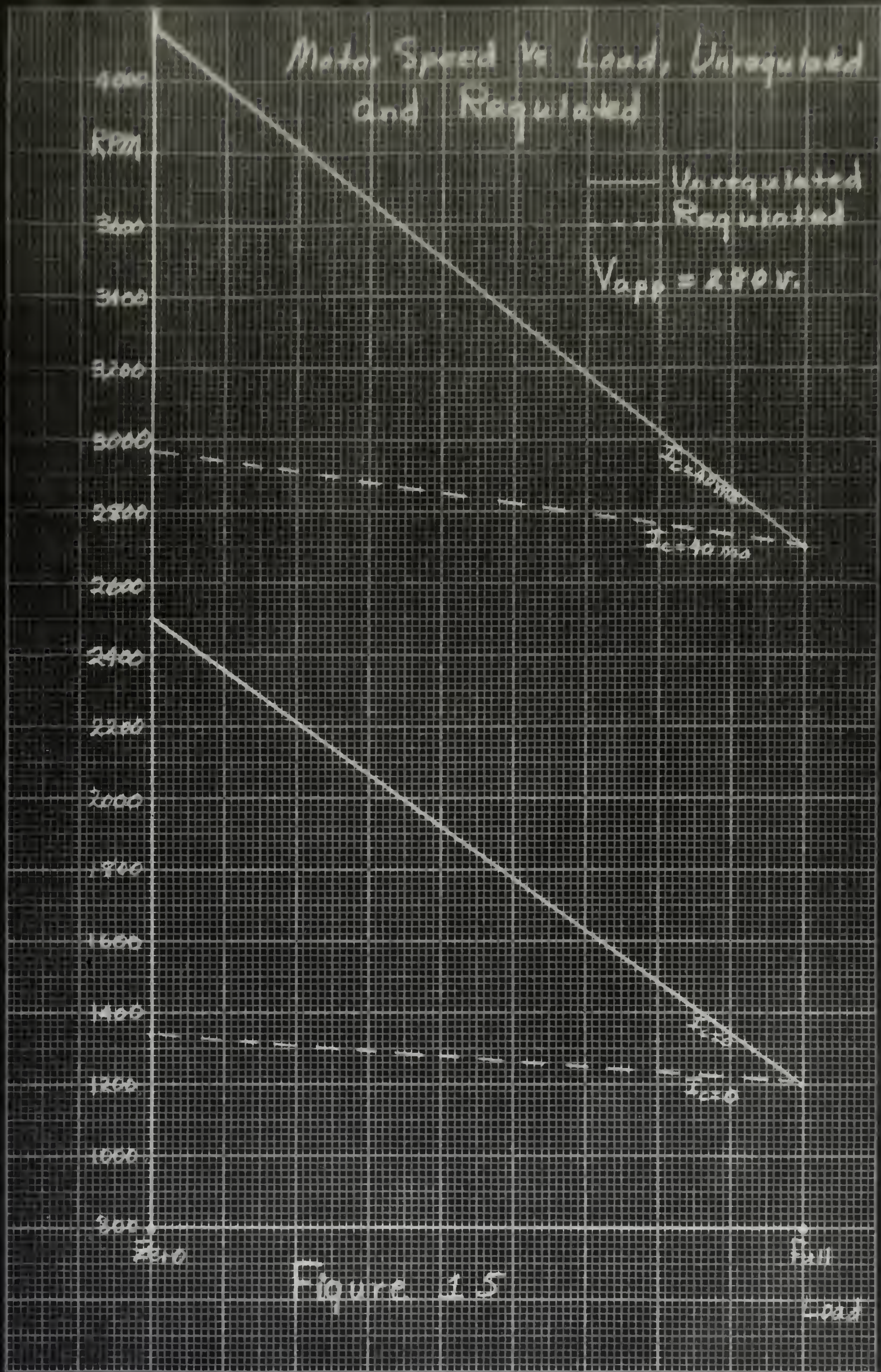
The loss of sensitivity due to the regulating winding may be computed by modifying equation (43) to include the magnetomotive force of the regulating winding.

$$(N I_{dc})_{total} = N_c I_c + \beta I_{load} N_{ac} - N_d I_d \qquad (47)$$

where $(N I_{dc})_{total}$ is the mmf required in the simple magnetic amplifier.

Then

$$I_c = \frac{(N I_{dc})_{total}}{N_c} - \frac{\beta I_{load} N_{ac}}{N_c} + \frac{N_d I_d}{N_c} \qquad (48)$$



Motor Speed vs Load, Unregulated and Regulated
Unregulated
Regulated
Vapp = 280 V.
RPM
4000
3600
3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
Zero
Full Load

Figure 15

For example, for zero load with motor speed equal to 2970 rpm,

$$I_c = 170 - \frac{1}{2}(320) + 20.7$$
$$= 30.7 \text{ ma.}$$

Actually, I_c was forty milliamperes. Note that the regulating winding appears to increase the control current by a considerable amount at zero load. I_c for this motor speed and zero load with no regulation was only about ten milliamperes. Regulation, then, more than doubled the amount of control current needed to produce a motor speed of 2970 rpm at no load. The decrease in sensitivity with regulation was, of course, maximum at no load. At full load there was no decrease in sensitivity when regulation was in effect since I_d was zero for full load.

9. Summary.

The circuit of Figure 13, it is felt, is a practical solution to the problem outlined in Section 1. The antenna could be rotated at any speed from zero to fifteen rpm under any load from zero to a sixty knot wind plus effect of inertial forces; the antenna speed was effectively regulated; and vacuum tubes were eliminated except as they might be used for regulating circuit similar to that of Figure 14.

BIBLIOGRAPHY

1. Boyajian, A. Mathematical analysis of non-linear circuits. General Electric review. 34: 531+, September 1931.

2. Dornhoefer, W. J. Self-saturating magnetic amplifiers. AIEE Technical Paper 49-140, May 1949.

3. Fitzgerald, A. S. Magnetic amplifier circuits-neutral type. Journal of the Franklin Institute. 244: 249-265, October 1947.

4. Fitzgerald, A. S. Some notes on the design of magnetic amplifiers. Journal of the Franklin Institute. 244: 323-362, November 1947.

5. Hughes, E. Magnetic characteristics of nickel-iron alloys with alternating magnetizing forces. Institution of electrical engineers. Journal. 79: 213-223, August 1936.

6. Logan, F. G. Saturable reactors and magnetic amplifiers. Electronics. 21: 104-109.

7. Morgan, R. E. The amplistat--a magnetic amplifier. American institute of electrical engineers, Journal. 68: 663-667, August 1949.

8. Rex, H. B. The transductor. Instruments. 20: 1102-1109, December 1947.

9. Summers, C. M. Mathematical expression of a saturation curve. General electric review. 36: 182-185, April 1933.

10. Ver Planck, D. W., Fishman, M., and Beaumariage, D. C. An analysis of magnetic amplifiers with feedback. Proceedings of the IRE. : 862-866, August 1940.

APPENDIX A

THE PROBLEM OF RADAR ANTENNA TRACKING

Chapter II has offered a magnetic amplifier solution to radar antenna control for searching. Present-day radars are also often used for tracking targets. It would be highly desirable, therefore, if a magnetic amplifier could be designed for this latter purpose. The circuit of Figure 1 illustrates the problem. When I_c is zero, the motor is stationary since there is as much current going through the motor in one way as the other. If the control current is now increased and the control windings are arranged so that the magnetomotive force of one control winding bucks that of its associated load winding while the magnetomotive force of the other control winding aids that of its associated load winding, then the load current should increase in one direction and decrease in the other, and the motor will turn in one direction. There are two things wrong with this idea. First, sensitivity is poor because there still is a current in the wrong direction through the motor. Second, such a control winding arrangement means that the load winding will induce voltages in the control windings. As was mentioned in Chapter I, this reduces the sensitivity. If the control current direction were reversed, the motor would turn in the opposite direction. This arrangement should work, but its sensitivity would be poor, and the motor would probably be sluggish in its response.

The control current would have to result from an error signal. This error signal, resulting from the antenna moving off the target,

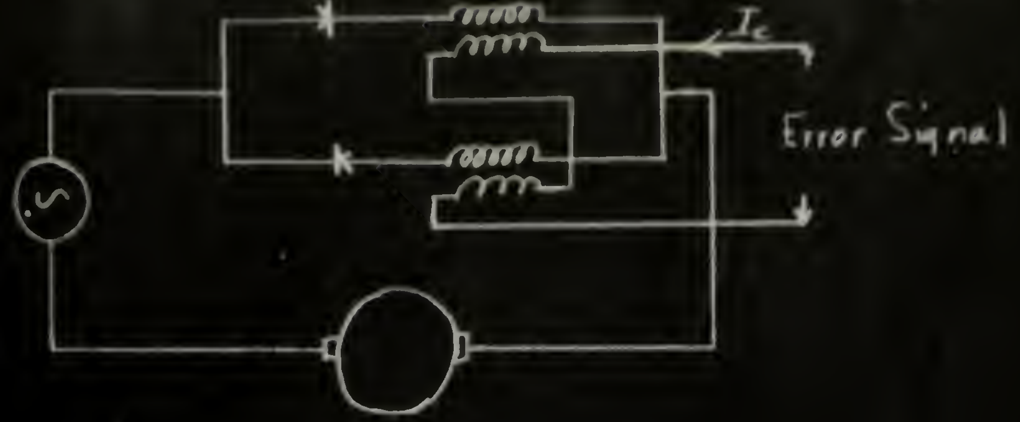


Figure 1 - First Tracking Circuit

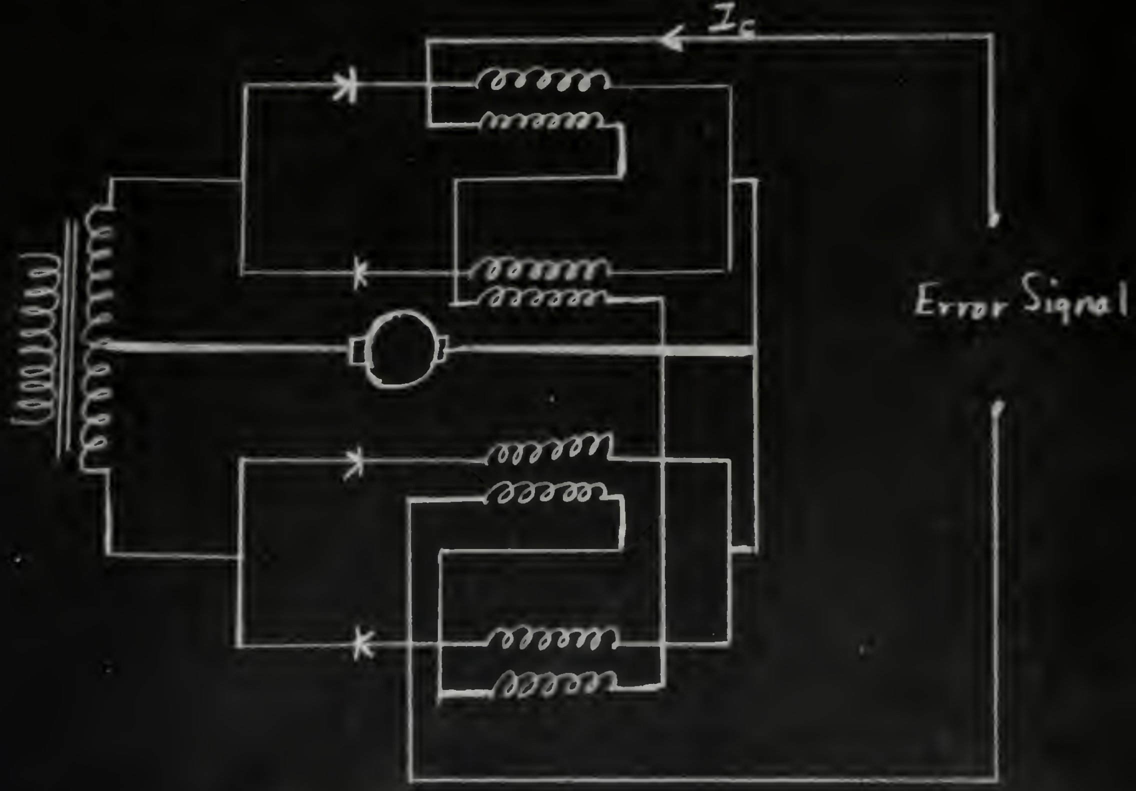


Figure 2 - Second Tracking Circuit

could be generated in the conventional way.

Figure 2 shows another circuit that will make the antenna track the target. When the top of the secondary is positive, current goes through the motor in one direction. When the polarity changes, current reverses through the motor. When I_c is zero, then, the motor is stationary. Due to the way the control windings are arranged, when I_c is increased, the current through the motor in one direction increases and it decreases in the other direction. Therefore the motor will turn. Reversal of the control current reverses the motor direction. The difficulty with this circuit is that not only is there a path for current from one pair of reactors through the motor, but there is also a path for this current through one of the other pair of reactors. This causes this circuit to be insensitive and requires inordinately large reactors for the amount of motor torque resulting.

The problem of antenna tracking using magnetic amplifiers will be solved when a circuit configuration is developed that will have good sensitivity, that will respond quickly to error, and will not cause hunting.

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